2023 58th International Scientific Conference on Information, Communication and Energy Systems and Technologies (ICEST) | 979-8-3503-1073-3/23/$31.00 ©2023 IEEE | DOI: 10.1109/ICEST58410.2023.10187293

Equivalences Induced by $(n,1)$, $(n,n-1)$ and (n,k) - Equivalences $(1<k<n-1)$

Sonja B. Chalamani[1], Marzanna J. Seweryn-Kuzmanovska[2] and Elena T. Kotevska[3]

Abstract – **In this paper we generate equivalences induced by $(n,1)$, $(n,n-1)$ and (n,k)-equivalences ($1<k<n-1$). The connections between such equivalences are proved. Moreover, properties of (n,m)- relations are investigated, and especially properties of relations induced from other relations, namely (n,m)- equivalence relations. Couple of examples are given for generalized equivalences.**

Keywords – (n,m)- **equivalence,** $(n,1)$, $(n,n-1)$ **and** (n,k)- **equivalence.**

I. Introduction

The notion of n-partition of set, introduced by Hartmanis, is closely connected with the notion of generalized equivalence relation that can be found in Pickett [1]. Several generalizations of the notion of equivalence relation are given in [2], [3] and [4]. In [5] the notion of (n,m)-equivalence relation is introduced. On the other hand, geometrical problems in metric spaces and their axiomatic classification are considered in [6], [7] and [8]. In [9] the notion of (n,m)-equivalence relation is used to introduce the notion of (n,m,ρ)- metrics, where ρ is an (n,m)- equivalence relation. In this paper the properties of (n,m)- relations are investigated, and specially properties of relations induced from other relations, namely (n,m)- equivalence relations. The relationships between (n,m)-equivalence relations and some characteristic properties are examined, for different n, m. (n,m)- equivalence relation is considered on finite and infinite sets and some examples are given. The generalization of equivalences is presented with four theorems. Moreover, equivalences induced by $(n,1)$, $(n,n-1)$ and (n,k) - equvalences $1<k<n-1$ are given.

[1]Sonja B. Chalamani is with University St Kliment Ohridski Bitola, Faculty of Technical Sciences, Makedonska Falanga 37, 7000 Bitola, North Macedonia, E-mail: sonja.calamani@uklo.edu.mk

[2]Marzanna J. Seweryn-Kuzmanovska is with University St Kliment Ohridski Bitola, Pedagogical Faculty, General Vasko Karangelevski B.B., 7000 Bitola, North Macedonia, E-mail: marzanna.kuzmanovska@yahoo.com

[3]Elena T. Kotevska is with University St Kliment Ohridski Bitola, Faculty of Technical Sciences, Makedonska Falanga 37, 7000Bitola, North Macedonia, E-mail: elena.kotevska@tfb.uklo.edu.mk

II. Definitions of (n,m)- Equivalences and Examples

Here we give some basic definitions and properties that are needed to introduce the new notions and to examine corresondent properties. First we give definitions for symmetric and reflexive relation on M, transitive (n,m)- relation on M and (n,m)-equivalence on M. Also, examples are given for these new notions.

Let n, m be two positive integers, such that $n-m=k\geq 1$ and let M be a nonempty set. With M^n we denote the n-th Cartesian product of M. The elements $\mathbf{x}\in M^n$ are sequences $(x_1,x_2,\ldots,x_n)$, $x_i\in M$. We denote the elements by $x_1x_2\ldots x_n$ or by $x_1{}^n$ and the sequence $(x,x,\ldots,x)\in M^n$ with $\overset{n}{x}$.

Definition 2.1. n - **th – permutation product** of M, that is n - **th symmetric product** of M is the set $M^{(n)}=M/\sim$, where $\sim$ is an equivalence relation defined on M^n with:

$$x_1^n\sim y_1^n \Leftrightarrow (x_1,\ldots,x_n)$$

is a permutation of $(y_1,\ldots,y_n)$.

We will use the same notation $\mathbf{x}=a_1^n$ for the elements in $M^{(n)}$ considering that $a_1^n=b_1^n$ in $M^{(n)}$ for $a_i,b_i\in M$ if and only if $(b_1,b_2,\ldots,b_n)$ is a permutation of $(a_1,a_2,\ldots,a_n)$.

Definition 2.2. A subset ρ of $M^{(n)}$ is called **symmetric** n - **relation** on M. Symmetric n - relation on M is called a **reflexive** n -**relation** on M if for each $a\in M$, $a^n=(a,\ldots,a)$ is in ρ. A symmetric n - relation on M is called a **transitive,** (n,m) **- relation** on M, i.e. (n,m) **- transitive**, if for each $\mathbf{x}\in M^{(n)}$ and each $\mathbf{b}\in M^{(m)}$, (s.t. $\mathbf{ub} \in \rho$ for each $\mathbf{u} \in M^{(k)}$, $n = m + k$, with $\mathbf{uv} = \mathbf{x}$) implies $\mathbf{x} \in \rho$.

A reflexive n - relation on M which is (n,m) - transitive is called (n,m) - **equivalence** on M.

Remark 2.1. With these notations $(2,1)$ – equivalence is the usual notation for equivalence.

Examples 2.1.

a) The set $\Delta = \{x^n \mid x \in M\}$ is (n,m) - equivalence on M for any $1 \le m < n$.

b) The set $\nabla = \{(x,x,y) \mid x,y \in M\}$ is $(3,1)$ - equivalence on M.

c) The set $\nabla_1 = \{(x,x,x,y) \mid x,y \in M\}$ is $(4,t)$ - equivalence on M for $t = 1,2$.

d) The set $\nabla_2 = \{(x,x,y,y) \mid x,y \in M\}$ is $(4,t)$ - equivalence on M for $t = 1,2,3$.

e) The set $\mathrm{Col} = \{(A,B,C) \mid A,B,C$ are collinear points in $E^2\}$ is $(3,1)$ - equivalence on E^2 where E^2 is the Euclidean plane.

f) The set $\mathrm{Com} = \{(A,B,C,D) \mid A,B,C,D$ are coplanar points in $E^3\}$ is $(4,1)$ - equivalence on E^3 where E^3 is the Euclidean 3 – dimensional space.

III. Equivalences induced by $(n,1)$, $(n,n-1)$ and (n,k) - equivalences $(1 < k < n-1)$

Let $\mathbf{x} = (x_1,\dots,x_{n+1}) \in M^{(n+1)}$ and let

$$\mathbf{x}\,(i) = (x_1,\dots,\ x_{i-1},\ x_{i+1},\dots,\ x_{n+1}) \in M^{(n)},$$

$$\mathbf{x}\,(i)(j) = \mathbf{x}\,(j)(i) = \mathbf{x}\,(i,j) \in M^{(n-1)} \quad \text{for} \quad i \ne j.$$

Proposition 3.1. Let $\rho \subseteq M^{(n)}$. If ρ is $(n,1)$ - equivalence on M, then $\psi(\rho) \subseteq M^{(n+1)}$ defined by

$$\mathbf{x} \in \psi(\rho) \Leftrightarrow \mathbf{x}\,(i) \in \rho \ \text{ for each } i \in \{1,2,\dots,n+1\} \qquad (1)$$

is $(n+1,1)$ and $(n+1,2)$ - equivalence on M.

Proof. 1^0 It is obviuous that for $\rho \subseteq M^{(n)}$, $\psi(\rho)$ is an $(n+1,1)$ - equivalence on M.

2^0 We will check whether $\psi(\rho)$ is $(n+1,2)$ - equivalence on M. Let $\mathbf{a} = (a_1,a_2)$ and let $\mathbf{ax}\,(i,j) \in \psi(\rho)$ for any $1 \le i \ne j \le n+1$. We will prove that $\mathbf{x} \in \psi(\rho)$.

Since $\mathbf{ax}\,(i,j) \in \psi(\rho)$ for any $1 \le i \ne j \le n+1$, it follows that $a_1\ \mathbf{x}\,(i,j) \in \rho$.

For fixed i_0 and for each $j \ne i_0$, $a_1\ \mathbf{x}\,(i_0,j) \in \rho$. Since ρ is $(n,1)$ - equivalence on M, we get that $\mathbf{x}\,(i_0) \in \rho$ for each $i_0 \in \{1,2,\dots,n+1\}$. According to definition (1), that means that $\mathbf{x} \in \psi(\rho)$. It follows $\psi(\rho)$ is $(n+1,2)$ - equivalence on M. ♦

Proposition 3.2. Let $\rho \subseteq M^{(n)}$. If ρ is $(n,n-1)$-equivalence on M, then $\psi(\rho) \subseteq M^{(n+1)}$ is $(n+1,t)$-equivalence on M for $1 < t \le n$.

In the proof of Proposition 3.2, we will use the following Proposition 3.3.

Proposition 3.3. Let $\rho \subseteq M^{(n)}$. If ρ is $(n,n-1)$-equivalence on M, then ρ is $(n,n-k)$- equivalence on M for $2 \le k \le n-1$.

Proof. Let ρ is $(n,n-1)$- equivalence on M. Let

$(a_1,\dots,a_{n-k},x_1,x_2,\dots,x_{k-1},x_k),(a_1,\dots,a_{n-k},x_1,x_2,\dots,x_{k-1},x_{k+1}),\dots,$

$(a_1,\dots,a_{n-k},x_1,x_2,\dots,x_{k-1},x_n),$

$\vdots$

$(a_1,\dots,a_{n-k},x_1,x_2,\dots,x_{k-2},x_k,x_{k+1}),$

$(a_1,\dots,a_{n-k},x_1,x_2,\dots,x_{k-2},x_k,x_{k+2}),\dots,$

$(a_1,\dots,a_{n-k},x_1,x_2,\dots,x_{k-2},x_k,x_n),$

$\vdots$

$(a_1,\dots,a_{n-k},x_2,x_3,\dots,x_{k-1},x_{k+1},x_{k+2}),$

$(a_1,\dots,a_{n-k},x_2,x_3,\dots,x_{k-1},x_{k+1},x_{k+3}),\dots,$

$(a_1,\dots,a_{n-k},x_2,x_3,\dots,x_{k-1},x_{k+1},x_n),$

$(a_1,\dots,a_{n-k},x_2,x_3,\dots,x_k,x_{k+2}),(a_1,\dots,a_{n-k},x_2,x_3,\dots,x_k,x_{k+3}),\dots,$

$(a_1,\dots,a_{n-k},x_2,x_3,\dots,x_{k-1},x_k,x_n),$

$\vdots$

$(a_1,\dots,a_{n-k},x_{n-k+1},x_{n-k+2},\dots,x_n) \in \rho.$

We want to check if $(\underbrace{x_1,x_2}_{n-k},\underbrace{\dots,x_n}_{k}) \in \rho$?

Since

$\underbrace{(a_1,\dots,a_{n-k},x_2,x_3,\dots,x_{k-1},x_{k+1},x_k),\dots,(a_1,\dots,a_{n-k},x_2,x_3,\dots,x_{k-1},x_{k+1},x_k)}_{n-1},$

$(a_1,\dots,a_{n-k},x_2,x_3,\dots,x_{k-1},x_{k+1},x_1) \in \rho$ and ρ is $(n,n-1)$-equivalence on M, we conclude that $(\underbrace{x_k,\dots,x_k}_{n-1},x_1) \in \rho$.

Analogously:

$\underbrace{(a_1,\dots,a_{n-k},x_1,x_3,\dots,x_{k-1},x_{k+1},x_k),\dots,(a_1,\dots,a_{n-k},x_1,x_3,\dots,x_{k-1},x_{k+1},x_k)}_{n-1},$

$$\left(a_1,\dots,a_{n-k},x_1,x_3,\dots,x_{k-1},x_{k+1},x_2\right)\in\rho\Rightarrow\left(\underbrace{x_k,\dots,x_k}_{n-1},x_2\right)\in\rho.$$

$\vdots$

$$\underbrace{\left(a_1,\dots,a_{n-k},x_1,x_2,\dots,x_{k-2},x_{k+1},x_k\right),\dots,\left(a_1,\dots,a_{n-k},x_1,x_2,\dots,x_{k-2},x_{k+1},x_k\right)}_{n-1},$$

$$\left(a_1,\dots,a_{n-k},x_1,x_2,\dots,x_{k-2},x_{k+1},x_{k-1}\right)\in\rho\Rightarrow\left(\underbrace{x_k,\dots,x_k}_{n-1},x_{k-1}\right)\in\rho.$$

$$\underbrace{\left(a_1,\dots,a_{n-k},x_2,x_3,\dots,x_{k-1},x_{k+2},x_k\right),\dots,\left(a_1,\dots,a_{n-k},x_2,x_3,\dots,x_{k-1},x_{k+2},x_k\right)}_{n-1},$$

$$\left(a_1,\dots,a_{n-k},x_2,x_3,\dots,x_{k-1},x_{k+2},x_{k+1}\right)\in\rho$$
$$\Rightarrow\left(\underbrace{x_k,\dots,x_k}_{n-1},x_{k+1}\right)\in\rho.$$

$\vdots$

$$\underbrace{\left(a_1,\dots,a_{n-k},x_1,x_2,\dots,x_{k-1},x_k\right),\dots,\left(a_1,\dots,a_{n-k},x_1,x_2,\dots,x_{k-1},x_k\right)}_{n-1},$$

$$\left(a_1,\dots,a_{n-k},x_1,x_2,\dots,x_{k-1},x_n\right)\in\rho\Rightarrow\left(\underbrace{x_k,\dots,x_k}_{n-1},x_n\right)\in\rho.$$

Since

$$\left(\underbrace{x_k,\dots,x_k}_{n-1},x_1\right),\ \left(\underbrace{x_k,\dots,x_k}_{n-1},x_2\right),\dots,\ \left(\underbrace{x_k,\dots,x_k}_{n-1},x_k\right),$$
$$\left(\underbrace{x_k,\dots,x_k}_{n-1},x_{k+1}\right),\dots,\ \left(\underbrace{x_k,\dots,x_k}_{n-1},x_n\right)\in\rho$$

and ρ is $(n,n-1)$-equivalence on M, it follows that $\left(x_1,x_2,\dots,x_n\right)\in\rho$. ♦

Proof of Proposition 3.2. We define $\psi(\rho)\subseteq M^{(n+1)}$ by (1).

Let $t=n$. We will check if $\psi(\rho)$ is $(n+1,n)$-equivalence on M. Let $\mathbf{a}=\left(a_1,\dots,a_n\right)$, $\mathbf{x}=\left(x_1,\dots,x_{n+1}\right)$ and let $\mathbf{a}\ x_{i}\in\psi(\rho)$ for each $i\in\{1,2,\dots,n+1\}$. We will prove that $\mathbf{x}\in\psi(\rho)$.

Since $x_{i}\in\psi(\rho)$ for each $i\in\{1,2,\dots,n+1\}$, we can conclude that $\mathbf{a}\ (n)x_i\in\rho$. But ρ is $(n,n-1)$- equivalence on M, so $\mathbf{x}\ (t)\in\rho$ for each $t\in\{1,2,\dots,n+1\}$ and by the definition (1) it means that $\mathbf{x}\in\psi(\rho)$. Consequently, $\psi(\rho)$ is $(n+1,n)$- equivalence on M. According to the Proposition 3.3., $\psi(\rho)$ is also $(n+1,t)$ - equivalence on M for $1\le t\le n-1$. ♦

Proposition 3.4. Let $\rho\subseteq M^{(n)}$. If ρ is (n,k)- equivalence on M $(1<k<n-1)$, then $\psi(\rho)\subseteq M^{(n+1)}$ is $(n+1,k-1)$, $(n+1,k)$ and $(n+1,k+1)$- equivalence on M.

Proof. We define $\psi(\rho)\subseteq M^{(n+1)}$ with (1).

1^0 We will check that $\psi(\rho)$ is $(n+1,k-1)$- equivalence on M. Let $\mathbf{a}=\left(a_1,\dots,a_{k-1}\right)$, $\mathbf{x}=\left(x_1,\dots,x_{n+1}\right)$ and let $\mathbf{ax}\ (i_1,\dots,i_{k-1})\in\psi(\rho)$ for each $1\le i_1<i_2<\dots<i_{k-1}\le n+1$. We will prove that $\mathbf{x}\in\psi(\rho)$.

Since
$$\mathbf{ax}\ (i_1,\dots,i_{k-1})\in\psi(\rho)$$
for each $1\le i_1<i_2<\dots<i_{k-1}\le n+1$, it follows that
$$\mathbf{ax}\ (j_1,\dots,j_k)\in\rho$$
for each $1\le j_1<j_2<\dots<j_k\le n+1$.

For fixed i_0, $\mathbf{ax}\ (j_1,\dots,j_k)=\mathbf{a}\ x_{i_0}\ \mathbf{x}\ (i_0,j_1,\dots,j_k)\in\rho$ for each $1\le j_1<j_2<\dots<j_k\le n+1$ and $j_s\ne i_0$. Since ρ is (n,k) - equivalence on M, we get that $\mathbf{x}\ (i_0)\in\rho$ for each $i_0\in\{1,2,\dots,n+1\}$. Then from the definition (1) we have that $\mathbf{x}\in\psi(\rho)$. It follows that $\psi(\rho)$ is $(n+1,k-1)$- equivalence on M.

2^0 Next, we will check if $\psi(\rho)$ is $(n+1,k)$- equivalence on M. Let $\mathbf{a}=\left(a_1,\dots,a_k\right)$, $\mathbf{x}=\left(x_1,\dots,x_{n+1}\right)$ and let $\mathbf{ax}\ (i_1,\dots,i_k)\in\psi(\rho)$ for each $1\le i_1<i_2<\dots<i_k\le n+1$. We will prove that $\mathbf{x}\in\psi(\rho)$.

Since
$$\mathbf{ax}\ (i_1,\dots,i_k)\in\psi(\rho)$$
for each $1\le i_1<i_2<\dots<i_k\le n+1$, it follows that for a fixed i_0, $\mathbf{ax}\ (i_0,j_1,\dots,j_k)\in\rho$ for each $1\le j_1<j_2<\dots<j_k\le n+1$ and $j_s\ne i_0$. But ρ is (n,k)- equivalence on M, so $\mathbf{x}\ (i_0)\in\rho$ for each $i_0\in\{1,2,\dots,n+1\}$. From the definition (1), $\mathbf{x}\in\psi(\rho)$. It follows that $\psi(\rho)$ is $(n+1,k)$- equivalence on M.

3^0 We will also check if $\psi(\rho)$ is $(n+1,k+1)$- equivalence on M.

Let $\mathbf{a}=\left(a_1,\dots,a_{k+1}\right)$, $\mathbf{b}=\left(a_1,\dots,a_k\right)$, $\mathbf{x}=\left(x_1,\dots,x_{n+1}\right)$ and let
$$\mathbf{ax}\ (i_1,\dots,i_{k+1})\in\psi(\rho)$$
for each $1\le i_1<i_2<\dots<i_{k-1}\le n+1$. We will prove that $\mathbf{x}\in\psi(\rho)$.

Since
$$\mathbf{ax}\ (i_1,\dots,i_{k+1})\in\psi(\rho)$$
for each $1\le i_1<i_2<\dots<i_k\le n+1$, it follows that
$$\mathbf{bx}\ (i_1,\dots,i_{k+1})\in\rho.$$

For fixed i_0, we define $\mathbf{y}_0=\mathbf{x}\ (i_0)$. Then, for each $1\le j_1<j_2<\dots<j_k\le n+1$ and $j_s\ne i_0$,
$$\mathbf{by}_0\ (j_1,\dots,j_k)=\mathbf{bx}\ (i_1,\dots,i_{k+1})\in\rho.$$

Since ρ is (n,k) - equivalence on M, we get that $\mathbf{y}_0 \in \rho$ for each $i_0 \in \{1,2,...,n+1\}$, i.e. $\mathbf{x}(i_0) \in \rho$ for each $i_0 \in \{1,2,...,n+1\}$. Then from the definition (1), $\mathbf{x} \in \psi(\rho)$. It follows that $\psi(\rho)$ is $(n+1,k+1)$ - equivalence on M. ♦

Example 3.1. Let $M = \{a,b,c,d\}$. Let

$$\rho = \Delta \cup \{(a,a,a,a,b),(a,a,a,a,c),(a,a,a,a,d),(b,b,b,b,a),(b,b,b,b,c),(b,b,b,b,d),(c,c,c,c,a),(c,c,c,c,b),(c,c,c,c,d),(a,a,a,b,b),(a,a,a,c,c),(b,b,b,a,a),(b,b,b,c,c),(c,c,c,a,a),(c,c,c,b,b),(a,a,a,b,c),(a,a,a,b,d),(a,a,a,c,d),(b,b,b,a,c),(b,b,b,a,d),(b,b,b,c,d),(c,c,c,a,b),(c,c,c,a,d),(c,c,c,b,d),(a,a,b,b,c),(a,a,b,b,d),(a,a,c,c,b)(a,a,c,c,d),(b,b,c,c,a),(b,b,c,c,d),(a,a,b,c,d),(b,b,a,c,d),(c,c,a,b,d)\}$$

be $(5,3)$ - equivalence on M. Then

$$\psi(\rho) = \Delta \cup \{(a,a,a,a,a,b),(a,a,a,a,a,c),(a,a,a,a,a,d),(b,b,b,b,b,a),(b,b,b,b,b,c),(b,b,b,b,b,d),(c,c,c,c,c,a),(c,c,c,c,c,b),(c,c,c,c,c,d),(a,a,a,a,b,b),(a,a,a,a,c,c),(b,b,b,b,a,a),(b,b,b,b,c,c),(c,c,c,c,a,a),(c,c,c,c,b,b),(a,a,a,a,b,c),(a,a,a,a,b,d),(a,a,a,a,c,d),(b,b,b,b,a,c),(b,b,b,b,a,d),(b,b,b,b,c,d),(c,c,c,c,a,b),(c,c,c,c,a,d),(c,c,c,c,b,d),(a,a,a,b,b,b),(a,a,a,c,c,c),(b,b,b,c,c,c),(a,a,a,b,b,c),(a,a,a,b,b,d),(a,a,a,c,c,b),(a,a,a,c,c,d),(b,b,b,a,a,c),(b,b,b,a,a,d),(b,b,b,c,c,a),(b,b,b,c,c,d),(c,c,c,a,a,b),(c,c,c,a,a,d),(c,c,c,b,b,a),(c,c,c,b,b,d),(a,a,a,b,c,d),(b,b,b,a,c,d),(c,c,c,a,b,d),(a,a,b,b,c,c),(a,a,b,b,c,d),(a,a,c,c,b,d),(b,b,c,c,a,d)\}$$

is $(6,1)$, $(6,2)$, $(6,3)$ and $(6,4)$ - equivalence on M, but it is not $(6,5)$ - equivalence on M, because

$$(a,a,a,a,a,a),(a,a,a,a,a,b),(a,a,a,a,a,b),(a,a,a,a,a,c),(a,a,a,a,a,d),(a,a,a,a,a,d) \in \psi(\rho),$$

but $(a,b,b,c,d,d) \notin \psi(\rho)$.

IV. Conclusion

In this paper the notion of (n,m) - equivalence is given as a generalization of equivalences. Also, presented equivalences are good basis for modern algebraic and computer sciences. Moreover, using these (n,m)-equivalences new metric and metrizable spaces can be generated, which can be widely used in game theory.

References

[1] E. Pickett, “A Note on Generalized Equivalence Relations“, Amer. Math. Monthly, no. 8, pp. 860-861, 1966.

[2] J. Ušan and B. Šešelja, “Transitive n-ary Relations and Characterisations of Generalized Equivalences“, Zbornik Rad. PMF Novi Sad, 11, pp. 231-234, 1981.

[3] J. Ušan, B. Šešelja, and G. Vojvodic, “General Ordering and Partitions“, Matenaticki vesnik, vol. 3, no. 16, pp. 241-247, 1979.

[4] J. Ušan and B. Šešelja, “On Some Generalizations of Reflexive, Antisymmetric and Transitive Relations“, Proc. of the Symp.n-ary structures, Skopje, pp. 175-183, 1982.

[5] D. Dimovski, “Generalized Metric—*(n,m,ρ)*-metrics“, Mat. Bilten, 16, Skopje, pp. 73-76, 1992.

[6] S. Gaehler, “2-Metrische Raume und ihre topologische Struktur“, Math. Nachr. 26, 1963, 115-148

[7] J. Ušan, “<E,Nm>-seti s (n+1)-rastojanjem“, Review of Research, PMF, Novi Sad, Ser. Mat.17, 2, pp. 65-87, 1989.

[8] S. Čalamani and D. Dimovski, “Topologies Induced by (3,1,ρ)-Metrics and (3,2,ρ)-Metrics“, Inernational Mathematical Forum, vol. 9, no. 21-24, pp. 1075-1088, 2014.

[9] D. Dimovski, “*(3,1,ρ)*-Metrizable Topological Spaces“, Math. Macedonica, 3, pp. 59-64, 2005.