Union of Mathematicians of Macedonia - ARMAGANKA

X SEMINAR OF DIFFERENTIAL EQUATIONS AND ANALYSIS

and

2nd CONGRESS OF DIFFERENTIAL EQUATIONS, MATHEMATICAL ANALYSIS AND APPLICATIONS

CODEMA 2022

Proceedings of the CODEMA 2022

Зборник на трудови од CODEMA 2022

Skopje, 2023

Уредник: Алекса Малчески
Илустратор: Ана Иванова
Печатница: Ацетони-Прилеп
CD електронска публикација
Тираж: 50

Union od Mathematicians of Macedonia – ARMAGANKA

Proceedings of the CODEMA 2022
Зборник трудови од CODEMA 2022

Skopje, 2023

CONTENTS

23. **SOLVING TASKS FROM LINEAR PROGRAMMING USING GEOGEBRA**

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ISBN 978-608-4904-04-5(електронско издание)
ISBN 978-608-4904-03-8(печатено издание)
UDC: 517.983.24

N-TUPLE WEAK ORBITS TENDING TO INFINITY FOR HILBERT SPACE OPERATORS

Sonja Mančevska, Marija Orovčanec

Abstract . In this paper we prove some results on the existence of a dense set of pairs in the direct product of an infinite-dimensional complex Hilbert space with itself such that each pair in this set has an *n*-tuple weak orbit tending to infinity for a specific countable family of mutually commuting bounded linear operators.

1. INTRODUCTION

For bounded linear operators on Banach spaces the concepts of *n*-tuple orbits and *n*-tuple weak orbits are defined as follows. If X is a complex and infinite-dimensional Banach space, $B(X)$ is the algebra of all bounded linear operators on X and $T_1, T_2, \ldots, T_n \in B(X)$ are mutually commuting operators, then the *n-tuple orbit* of the vector $x \in X$ is the set

$$\mathrm{Orb}(\{T_i\}_{i=1}^{n}, x) = \left\{ T_1^{k_1} T_2^{k_2} \ldots T_n^{k_n} x : k_i \geq 0; 1 \leq i \leq n \right\}. \tag{1.1}$$

The *n*-tuple orbit *tends to infinity* if

$$\lim_{k_i \to \infty} \left\| T_1^{k_1} T_2^{k_2} \ldots T_n^{k_n} x \right\| = \infty \text{, for all } k_j \geq 0 \text{, } j \neq i \text{, } 1 \leq i, j \leq n \text{ .}$$

For $n = 1$, the *n*-tuple orbit (1.1) reduces to a simple sequence of form

$$\mathrm{Orb}(T, x) = \left\{ T^n x : n = 0, 1, 2, \ldots \right\},$$

usually referred as *single orbit* (or simply *orbit*) of the vector $x \in X$ under the operator T . If X^* is the dual space of X , i.e., the space of all bounded linear functionals $x^* : X \to \mathbb{C}$, and for $x \in X$ and $x^* \in X^*$, $\langle x, x^* \rangle := x^*(x)$, the *n-tuple weak orbit* of the pair $(x, x^*) \in X \times X^*$ is a set of form

$$\mathrm{Orb}(\{T_i\}_{i=1}^{n}, x, x^*) = \left\{ \left\langle T_1^{k_1} T_2^{k_2} \ldots T_n^{k_n} x, x^* \right\rangle : k_i \geq 0; 1 \leq i \leq n \right\}. \tag{1.2}$$

The *n*-tuple weak orbit *tends to infinity* if

2010 *Mathematics Subject Classification.* Primary: 47A05. Secondary: 47A11, 47A25.
Key words and phrases. Hilbert spaces, weak orbits, n-tuple weak orbits, sequences of operators.

$$\lim_{k_i \to \infty} \left| \left\langle T_1^{k_1} T_2^{k_2} \ldots T_n^{k_n} x, x^* \right\rangle \right| = \infty \text{, for all } k_j \geq 0 \text{, } j \neq i \text{, } 1 \leq i, j \leq n \text{ .}$$

For $n=1$, the *n*-tuple weak orbit (1.2) reduces to a simple scalar sequence of form

$$\mathrm{Orb}(T, x, x^*) = \left\{ \left\langle T^n x, x^* \right\rangle : n = 0,1,2,\ldots \right\},$$

usually referred as *weak orbit* of the pair $(x, x^*) \in X \times X^*$ under the operator T .

For the case of Hilbert spaces, by the Riesz Theorem for representation of a bounded linear functional on Hilbert spaces (cf. [7,III.6]), given an infinite-dimensional complex Hilbert space H with an inner product $\langle \cdot | \cdot \rangle$, its dual space H^* can be fully identified with the space itself since

$$H^* = \left\{ x \mapsto \langle x | y \rangle, x \in H : y \in H \right\} .$$

Hence, for a set of mutually commuting operators $T_1, T_2, \ldots, T_n \in B(H)$ the *n*-tuple weak orbits will be the sets of form

$$\mathrm{Orb}(\{T_i\}_{i=1}^n, x, y) = \left\{ \left\langle T_1^{k_1} T_2^{k_2} \ldots T_n^{k_n} x \middle| y \right\rangle : k_i \geq 0; 1 \leq i \leq n \right\},\ (x, y) \in H \times H \text{ .}$$

In this paper we will consider only the conditions under which the direct product $H \times H$ contains a dense of pairs (x, y) with *n*-tuple weak orbits tending to infinity that do not involve any requirements upon specific subsets of the spectra of the operators. For $H \times H$ we will assume that is endowed with the product topology. Given an operator $T \in B(H)$, $\sigma(T)$ and $r(T)$ will denote the spectrum and the spectral radius of the operator T , respectively.

2. PRELIMINARY RESULTS

Theorem 2.1. ([6, Theorem V.39.8]) *Let H and K be Hilbert spaces, $(T_n)_{n\geq 1}$ be a sequence of operators in $B(H, K)$ and $(a_n)_{n\geq 1}$ be sequence of positive numbers with $\sum_{n\geq 1} a_n < \infty$. Then*

(i) *there are $x \in H$ and $y \in K$ such that and $\left| \langle T_n x | y \rangle \right| \geq a_n \left\| T_n \right\|$, for all n;*

(ii) *there is a dense subset of pairs $(x, y) \in H \times K$ such that $\left| \langle T_n x | y \rangle \right| \geq a_n \left\| T_n \right\|$, for all n sufficiently large.*

Corollary 2.2. ([6, Corollary V.39.9]) *Let H be Hilbert space and $T \in B(H)$ is such that $\sum_{k=1}^{\infty} \left\| T^k \right\|^{-1} < \infty$. Then there exist $x, y \in H$ such that $\left| \langle T^n x | y \rangle \right| \to \infty$. Moreover, the set of such pairs (x, y) is dense in $H \times H$.*

Lemma 2.3. ([6, Lemma V.37.15]) *Let* $\varepsilon > 0$ *and* $(a_n)_{n\geq 1}$ *be a sequence of positive numbers satisfying* $\sum_{n\geq 1} a_n < \varepsilon$. *Then there is a sequence of positive numbers* $(b_n)_{n\geq 1}$ *such that* $b_n \to \infty$ *as* $n \to \infty$ *and* $\sum_{n\geq 1} a_n b_n < \varepsilon$.

3. MAIN RESULTS

Let $F = \{1, 2, \ldots, N\}$ for some $N \in \mathbb{N}$, $N \geq 2$, or $F = \mathbb{N}$.

Theorem 3.1. *Let* H *be a Hilbert space,* $\{T_i : i \in F\} \subset B(H)$ *and* $\{(a_{i,j})_{j\geq 1} : i \in F\}$ *be a family of sequences of positive numbers such that* $\sum_{i\in F, j\geq 1} a_{i,j} < \infty$. *Then for any open balls* B_1 *and* B_2 *in* H *there are vectors* $x \in B_1$, $y \in B_2$ *and* $k_0 \in \mathbb{N}$ *such that*

$$\left|\left\langle T_i^k x \middle| y \right\rangle\right| \geq a_{i,k} \left\|T_i^k\right\|, \text{ for all } i \in F \text{ and } k \geq k_0.$$

Proof. Let $T_{i,k} := T_i^k$ ($i \in F$, $k \in \mathbb{N}$), $f : F \times \mathbb{N} \to \mathbb{N}$ be the bijective mapping defined with

$$f(i,j) = \begin{cases} i + N(j-1), & \text{if } F = \{1, 2, \ldots, N\} \\ \dfrac{(i+j-2)(i+j-1)}{2} + j, & \text{if } F = \mathbb{N} \end{cases},$$

and let $g : \mathbb{N} \to F \times \mathbb{N}$ denote its inverse mapping. If $(a'_n)_{n\geq 1}$ is a sequence of positive numbers and $(T'_n)_{n\geq 1}$ is a sequence of operators defined with

$$a'_n = a_{g(n)} \text{ and } T'_n = T_{g(n)}, \text{ for all } n \in \mathbb{N},$$

then $\sum_{n\geq 1} a'_n = \sum_{i\in F, j\geq 1} a_{i,j} < \infty$. Hence (by Theorem 2.1. (ii), applied on $(a'_n)_{n\geq 1}$, $(T'_n)_{n\geq 1}$ and $H = K$), if B_1 and B_2 are open balls H, then there are $x \in B_1$, $y \in B_2$ and $n_0 \in \mathbb{N}$ such that

$$\left|\left\langle T'_n x \middle| y \right\rangle\right| \geq a'_n \left\|T'_n\right\|, \text{ for all } n \geq n_0. \tag{3.1}$$

Since $f : F \times \mathbb{N} \to \mathbb{N}$ is bijective, there is a unique pair $(i_0, j_0) \in F \times \mathbb{N}$ such that $n_0 = f(i_0, j_0)$. Let

$$k_0 = \begin{cases} j_0 + 1, & \text{if } F = \{1, 2, \ldots, N\} \\ i_0 + j_0, & \text{if } F = \mathbb{N} \end{cases}.$$

If $(i,k) \in F \times \mathbb{N}$ is such that $k \geq k_0$, then by the definition of $f : F \times \mathbb{N} \to \mathbb{N}$ we have:

1. for $F = \{1, 2, \ldots, N\}$,

$$f(i,k) = i + N(k-1) \geq N(k_0 - 1) = Nj_0 = N + N(j_0 - 1)$$
$$\geq i_0 + N(j_0 - 1) = n_0 ,$$

2. for $F = \mathbb{N}$,

$$f(i,k) = \frac{(i+k-2)(i+k-1)}{2} + k \geq \frac{(i_0 + j_0 - 2)(i_0 + j_0 - 1)}{2} + j_0 = n_0.$$

Hence, by (3.1) and the definition of $(a'_n)_{n\geq 1}$ and $(T'_n)_{n\geq 1}$ we obtain

$$\left|\left\langle T_i^k x \middle| y \right\rangle\right| = \left|\left\langle T_{i,k} x \middle| y \right\rangle\right| = \left|\left\langle T_{g(n)} x \middle| y \right\rangle\right| \geq a_{g(n)} \left\| T_{g(n)} \right\| = a_{i,k} \left\| T_i^k \right\|,$$

for all $i \in F$ and $k \geq k_0$. ■

Theorem 3.2. *If H is Hilbert space and $\{T_i : i \in F\} \subset B(H)$ is a family of operators such that $\sum_{k=1}^{\infty} \left\| T_i^k \right\|^{-1} < \infty$, for all $i \in F$, then there is a dense set $D \subset H \times H$ such that the weak orbit $\left(\left\langle T_i^k x \middle| y \right\rangle\right)_{k\geq 0}$ tends to infinity for every pair $(x,y) \in D$ and every $i \in F$. If, in addition, $\{T_i : i \in F\}$ is a family of mutually commuting operators such that the sequence $(T_i^k - T_j^k)_{k\geq 1}$ is norm bounded for all $i,j \in F$, then for every $n \in F$ and $1 < m \leq n$, the $m-$tuple weak orbit*

$$\left\{\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} \ldots T_{i_m}^{k_m} x \middle| y \right\rangle : k_i \geq 0; 1 \leq i \leq m \right\},$$

tends to infinity for all $1 \leq i_1 < i_2 < \ldots < i_m \leq n$.

Proof. Let B_1 and B_2 be open balls H. For $i \in F$, let $\varepsilon_i > 0$ be such that

$$\varepsilon_i \left(\sum_{k=1}^{\infty} \frac{1}{\left\| T_i^k \right\|} \right) < \frac{1}{2^{i+1}},$$

and (by Lemma 2.3) let $(b_{i,k})_{k\geq 1}$ be the sequence of positive numbers such that $b_{i,k} \to \infty$ as $k \to \infty$ and

$$\sum_{k=1}^{\infty} \frac{\varepsilon_i b_{i,k}}{\left\| T_i^k \right\|} < \frac{1}{2^{i+1}}. \tag{3.2}$$

If $a_{i,k} = \varepsilon_i b_{i,k} \left\| T_i^k \right\|^{-1}$, $(i,k) \in F \times \mathbb{N}$, then by (3.2) we have

$$\sum_{i\in F, k\geq 1} a_{i,k} = \sum_{i\in F} \sum_{k=1}^{\infty} \frac{\varepsilon_i b_{i,k}}{\left\| T_i^k \right\|} < \sum_{i\in F} \frac{1}{2^{i+1}} < \frac{1}{2}.$$

Hence, by Theorem 3.1, there are $x \in B_1$, $y \in B_2$ and $k_0 \in \mathbb{N}$ such that

$\left|\left\langle T_i^k x \middle| y \right\rangle\right| \geq a_{i,k} \left\| T_i^k \right\| = \varepsilon_i b_{i,k} \left\| T_i^k \right\|^{-1} \left\| T_i^k \right\| = \varepsilon_i b_{i,k}$, for all $i \in F$ and $k \geq k_0$.
Letting $k \to \infty$, we have

$$\lim_{k \to \infty} \left|\left\langle T_i^k x \middle| y \right\rangle\right| = \infty \text{, for all } i \in F. \tag{3.3}$$

If, in addition, $\{T_i : i \in F\}$ is a family of mutually commuting operators such that the sequence $(T_i^k - T_j^k)_{k \geq 1}$ is norm bounded for all $i, j \in F$, let $M_{i,j} > 0$ is such that $\left\| T_i^k - T_j^k \right\| \leq M_{i,j}$, for all $k \geq 0$, and let $(x, y) \in H \times H$ be a pair satisfying (3.3). We continue by induction.

Let $m = 2$ and $1 \leq i_1 < i_2 \leq n$. By the Cauchy-Bunyakovsky-Schwarz inequality we have

$$\begin{aligned}
\left|\left\langle T_{i_1}^{k_1+k_2} x \middle| y \right\rangle\right| &\leq \left|\left\langle T_{i_1}^{k_1+k_2} x - T_{i_1}^{k_1} T_{i_2}^{k_2} x \middle| y \right\rangle\right| + \left|\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} x \middle| y \right\rangle\right| \\
&= \left|\left\langle T_{i_1}^{k_1} (T_{i_1}^{k_2} - T_{i_2}^{k_2}) x \middle| y \right\rangle\right| + \left|\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} x \middle| y \right\rangle\right| \\
&\leq \left\| T_{i_1}^{k_1} (T_{i_1}^{k_2} - T_{i_2}^{k_2}) x \right\| \cdot \|y\| + \left|\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} x \middle| y \right\rangle\right| \\
&\leq \left\| T_{i_1}^{k_1} \right\| \cdot \left\| T_{i_1}^{k_2} - T_{i_2}^{k_2} \right\| \cdot \|x\| \cdot \|y\| + \left|\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} x \middle| y \right\rangle\right| \\
&\leq \left\| T_{i_1} \right\|^{k_1} \cdot M_{i_1,i_2} \cdot \|x\| \cdot \|y\| + \left|\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} x \middle| y \right\rangle\right|.
\end{aligned}$$

Since $\left|\left\langle T_{i_1}^n x \middle| y \right\rangle\right| \to \infty$ as $n \to \infty$ (hence $\left|\left\langle T_{i_1}^{k_1+k_2} x \middle| y \right\rangle\right| \to \infty$ as $k_2 \to \infty$, for all $k_1 \geq 0$), the above inequalities imply that

$$\left|\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} x \middle| y \right\rangle\right| \to \infty \text{, as } k_2 \to \infty \text{, for all } k_1 \geq 0.$$

Similarly,

$$\begin{aligned}
\left|\left\langle T_{i_2}^{k_1+k_2} x \middle| y \right\rangle\right| &\leq \left|\left\langle T_{i_2}^{k_1+k_2} x - T_{i_1}^{k_1} T_{i_2}^{k_2} x \middle| y \right\rangle\right| + \left|\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} x \middle| y \right\rangle\right| \\
&= \left|\left\langle T_{i_2}^{k_2} (T_{i_2}^{k_1} - T_{i_1}^{k_1}) x \middle| y \right\rangle\right| + \left|\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} x \middle| y \right\rangle\right| \\
&\leq \left\| T_{i_2} \right\|^{k_2} \cdot M_{i_2,i_1} \cdot \|x\| \cdot \|y\| + \left|\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} x \middle| y \right\rangle\right|,
\end{aligned}$$

which implies that

$$\left|\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} x \middle| y \right\rangle\right| \to \infty \text{, as } k_1 \to \infty \text{, for all } k_2 \geq 0.$$

To complete the proof, it is enough to show the claim is true for $m = n$, under the assumption that the (n-1)-tuple weak orbit

$$\left\{\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} \dots T_{i_{n-1}}^{k_{n-1}} x \middle| y \right\rangle : k_j \geq 0; 1 \leq j \leq n-1 \right\},$$

tends to infinity for all $1 \leq i_1 < \dots < i_{n-1} \leq n$. For a fixed $i \in \{1,2,\dots,n\}$, arbitrary $j \in \{1,2,\dots,n\} \setminus \{i\}$ and fixed $k_1, k_2, \dots, k_n \geq 0$ we have

$$\begin{aligned}
&\left|\left\langle T_1^{k_1} \dots T_{i-1}^{k_{i-1}} T_j^{k_i} T_{i+1}^{k_{i+1}} \dots T_n^{k_n} x \middle| y \right\rangle\right| \\
&\leq \left|\left\langle T_1^{k_1} \dots T_{i-1}^{k_{i-1}} T_j^{k_i} T_{i+1}^{k_{i+1}} \dots T_n^{k_n} x - T_1^{k_1} T_2^{k_2} \dots T_n^{k_n} x \middle| y \right\rangle\right| + \left|\left\langle T_1^{k_1} T_2^{k_2} \dots T_n^{k_n} x \middle| y \right\rangle\right| \\
&= \left|\left\langle T_1^{k_1} \dots T_{i-1}^{k_{i-1}} T_{i+1}^{k_{i+1}} \dots T_n^{k_n} (T_j^{k_i} - T_i^{k_i}) x \middle| y \right\rangle\right| + \left|\left\langle T_1^{k_1} T_2^{k_2} \dots T_n^{k_n} x \middle| y \right\rangle\right| \\
&\leq \left\| T_1^{k_1} \dots T_{i-1}^{k_{i-1}} T_{i+1}^{k_{i+1}} \dots T_n^{k_n} (T_j^{k_i} - T_i^{k_i}) x \right\| \cdot \|y\| + \left|\left\langle T_1^{k_1} T_2^{k_2} \dots T_n^{k_n} x \middle| y \right\rangle\right| \\
&\leq \left(\prod_{\substack{l=1 \\ l \neq i}}^{n} \|T_l\|^{k_l} \right) \cdot M_{i,j} \cdot \|x\| \cdot \|y\| + \left|\left\langle T_1^{k_1} T_2^{k_2} \dots T_n^{k_n} x \middle| y \right\rangle\right|.
\end{aligned}$$

Since $j \in \{1,2,\dots,n\} \setminus \{i\}$, by the inductive assumption, we have

$$\left|\left\langle T_1^{k_1} \dots T_{i-1}^{k_{i-1}} T_j^{k_i} T_{i+1}^{k_{i+1}} \dots T_n^{k_n} x \middle| y \right\rangle\right| \to \infty \text{ as } k_i \to \infty, \text{ for all } k_j \geq 0, \ j \neq i.$$

This, together with the above inequalities implies that

$$\left|\left\langle T_1^{k_1} T_2^{k_2} \dots T_n^{k_n} x \middle| y \right\rangle\right| \to \infty \text{ as } k_i \to \infty, \text{ for all } k_j \geq 0, \ j \neq i,$$

which completes the proof. ■

Corollary 3.3. *If H is a Hilbert space and $\{T_i : i \in F\} \subset B(H)$ is a family of operators such that $r(T_i) > 1$, for all $i \in F$, then there is a dense set $D \subset H \times H$ such that the weak orbit $\left(\left\langle T_i^k x \middle| y \right\rangle\right)_{k \geq 0}$ tends to infinity for every pair $(x, y) \in D$ and every $i \in F$. If, in addition, $\{T_i : i \in F\}$ is a family of mutually commuting operators such that the sequence $(T_i^k - T_j^k)_{k \geq 1}$ is norm bounded for all $i, j \in F$, then for every $n \in F$, every $1 < m \leq n$ the $m-$tuple weak orbit*

$$\left\{\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} \dots T_{i_m}^{k_m} x \middle| y \right\rangle : k_i \geq 0; 1 \leq i \leq m \right\},$$

tends to infinity for all $1 \leq i_1 < i_2 < \dots < i_m \leq n$.

Proof. If $T \in B(H)$ has a spectral radius $r(T) > 1$, then $\sum_{k=1}^{\infty} \left\| T^k \right\|^{-1} < \infty$. Namely, if $r(T) > 1$, then there is $\lambda \in \sigma(T)$ such that $1 < |\lambda|$. By the Spectral Mapping Theorem, $\lambda^n \in \sigma(T^n)$ for every $n \in \mathbb{N}$. Hence $|\lambda|^n \leq r(T^n) \leq \|T^n\|$ and

$$\sum_{n=1}^{\infty}\frac{1}{\left\|T^n\right\|}\leq\sum_{n=1}^{\infty}\frac{1}{|\lambda|^n}<\infty .$$

Now the conclusion follows from Theorem 3.2. ■

4. REMARKS ON N-TUPLE ORBITS TENDING TO INFINITY

By the Cauchy-Bunyakovsky-Schwarz inequality we have

$$\left|\left\langle T_1^{k_1}T_2^{k_2}\dots T_n^{k_n}x\middle|y\right\rangle\right|\leq\left\|T_1^{k_1}T_2^{k_2}\dots T_n^{k_n}x\right\|\cdot\left\|y\right\|,$$

for all $(x,y)\in H\times H$, $k_j\geq 0$ and $1\leq j\leq n$. These inequalities clearly imply that the *n*-tuple orbit $\mathrm{Orb}(\{T_i\}_{i=1}^n,x)$ tends to infinity whenever there is $y\in H$ such that the *n*-tuple weak orbit $\left\{\left\langle T_1^{k_1}T_2^{k_2}\dots T_n^{k_n}x\middle|y\right\rangle : k_i\geq 0; 1\leq i\leq n\right\}$ tends to infinity. Hence, from the results in the previous section we can derive the following results for *n*-tuple orbits tending to infinity.

Theorem 4.1. *If* H *is Hilbert space and* $\{T_i : i\in F\}\subset B(H)$ *is a family of operators such that* $\sum_{k=1}^{\infty}\left\|T_i^k\right\|^{-1}<\infty$ *for all* $i\in F$ *, then there is a dense set* $D\subset H$ *such that the orbit* $\mathrm{Orb}(T_i,x)$ *tends to infinity for every* $x\in D$ *and every* $i\in F$ *. If, in addition,* $\{T_i : i\in F\}$ *is a family of mutually commuting operators such that the sequence* $(T_i^k-T_j^k)_{k\geq 1}$ *is norm bounded for all* $i,j\in F$ *, then for every* $n\in F$ *, every* $1<m\leq n$ *, the* $m-$*tuple orbit* $\mathrm{Orb}(\{T_{i_j}\}_{j=1}^m,x)$ *tends to infinity for all* $1\leq i_1<i_2<\ldots<i_m\leq n$ *.*

Corollary 4.2. *If* H *is Hilbert space and* $\{T_i : i\in F\}\subset B(H)$ *is a family of operators such that* $r(T_i)>1$ *for all* $i\in F$ *, then there is a dense set* $D\subset H$ *such that the orbit* $\mathrm{Orb}(T_i,x)$ *tends to infinity or every* $x\in D$ *and every* $i\in F$ *. If, in addition,* $\{T_i : i\in F\}$ *is a family of mutually commuting operators such that the sequence* $(T_i^k-T_j^k)_{k\geq 1}$ *is norm bounded for all* $i,j\in F$ *, then for every* $n\in F$ *, every* $1<m\leq n$ *, the* $m-$*tuple orbit* $\mathrm{Orb}(\{T_{i_j}\}_{j=1}^m,x)$ *tends to infinity for all* $1\leq i_1<i_2<\ldots<i_m\leq n$ *.*

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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