

**Service Sector In Terms Of Changing Environment**

**ANALYSIS OF MORTALITY IN REPUBLIC OF  
MACEDONIA**

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**Abstract**

Calculations of the life insurance premium depends on values in mortality tables. Mortality tables can be modeled by using different functions for mortality analysis. Mortality tables give different values for both genders, and they differ from one decade to the following one. Those differences can very distinctive. Hence, the insurers should use the updated mortality tables and forecasting of future values of mortality. There are many different models of identification of mortality functions and models for forecasting. In this paper we use Azbel model and possibilities it gives for identification and forecasting future values of mortality. The Azbel model is also compared with time series analysis model. Data are taken for the mortality in Republic of Macedonia.

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"Keywords: : mortality, tables, identification, model;"

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**1. Introduction**

(10 pt) Here introduce the paper, and put a nomenclature if necessary, in a box with the same font size as the rest of the paper. The paragraphs continue from here and are only separated by headings, subheadings, images and formulae. The section headings are arranged by numbers, bold and 10 pt. Here follows further instructions for authors.

One and the most important parameter in calculation life insurance premium is current and future values of mortality [2]. Insurance models for premium calculation take the mortality as constant value during the years. [1]. Furthermore, some insurance companies uses mortality tables that are more than 10 year old, and they don't present the actual mortality values. Identification and forecasting future mortality values can be also applicable for calculation of retirement fees. Those calculations should not be accurate if we use old, or inappropriate mortality tables..

This problem is even more challenging in the countries of EU. Some new members has caused the change in mortality tables, and we can expect that changes should continue to be such a problem in the future also.

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In this paper we have analyzed mortality tables in R. Macedonia for the period 1966-2009. We have calculated the survival probability for some age values and forecasting the future mortality values. As a model for identification we have used the Azbel model. It's one of the simplest models for mortality identification, and it gives very good results in modeling mortality. The procedure of identification and forecasting is the same for any other country. Mortality data are taken from the Bureau of statistics in R. Macedonia. As additional source of mortality tables one can use the database of table from Society of actuaries, where we can find many mortality tables for different countries.

(<http://www.soa.org/ccm/content/areas-of-practice/special-interest-sections/computer-science/table-manager/>).

In the following text we have made mortality analysis by using modified Azbel model. The time series of mortality in Macedonia are also identified with time series analysis. At the end we have compared both models of identification.

**2. Analysis of mortality tables**

To consider the need for updated data, we should analyze the differences between mortality tables from past and recent values of mortality tables. On fig. 1 are presented graphs of mortality tables for the first and last analyzed year, 1966 and 2009.

From the fig. 1 we can identify obvious differences between mortality values in 1966 and 2009. The values of mortality percent for the age interval 15-44 are higher in the table of mortality in 1966. From the same figure, it is obvious that the mortality for the ages 65-84 is much higher in mortality table of 2009, than in 1966.

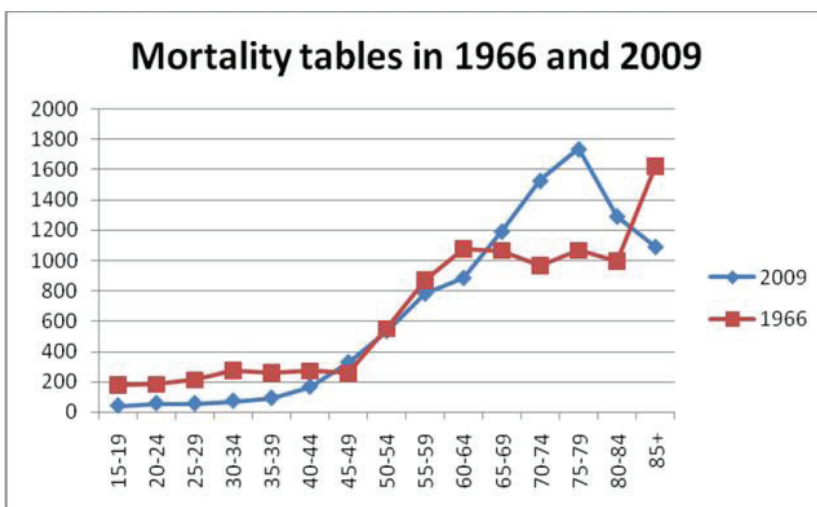


Figure 1. Mortality tables for the years 1966 and 2009

The values of mortality are normalized to count the same number of analyzed people in 1966 and 2004. On fig. 2 it is presented the values of mortality in R. Macedonia for the age of 15, and another figure for the ages of 70-74, for the whole set of mortality tables 1966-2009.

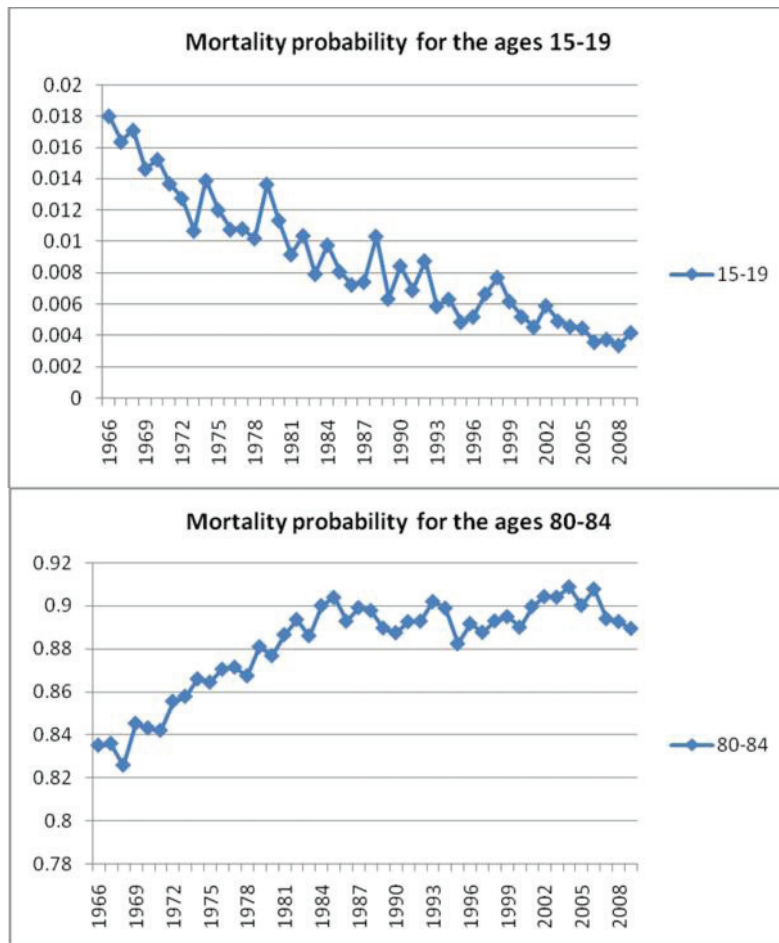


Fig. 2. Mortality values for the ages 15-19 and 70-74

Data presented in fig. 2 confirms the previous conclusions, that during the years (since 1966 till 2009) the number of deaths for the age of 45 is decreasing, and on the other hand the number of deaths for the ages greater than 60 is increasing, for the same number of analyzed people. For the taken samples for ages between 15 and 19, the mortality is decreasing from 1.802757% to 0.4349%. For the ages between 70 and 74, the mortality is increasing from the value of 9.80912% in 1966 to 16.199% in 2009.

From fig 2 we can see that at the beginning of 90-s the mortality value for the ages 70-74 is decreasing. The cause of this changes should be search in transitional processes of the country, loses of jobs and stability. If we want to try to model this time series we should involve dummy variable for this period.

There are many function models for modeling mortality function. These functions can be used for identification and forecasting of mortality. Starting from Gompertz model, through Lee-Carter till the Azbel model of mortality function we can find many different models, all based on exponential function. In this paper for modeling and forecasting mortality in R. Macedonia we use the simplest but efficient, Azbel model.

### 3. Azbel model for mortality function

The Azbel model for identification of mortality, just like Gompertz and Makeham model can be used for modeling mortality. It's given by following equation:

$$q_x = Ab \exp[b(x-X)] \quad (1)$$

where A, b and X are parameters. This formula can be further simplified if we include some replacements. Let  $T=X-(1/b)\ln(Ab)$ , then we can get the following formula:

$$q_x = \exp[b(x-T)] \quad (2)$$

The parameters from the formula (2) have the following roles: b determines the inclination of the function, T is the final age of the mortality table.  $q_x$  takes values close to 1 when x is close to the value of T.

For calculation of optimal value of parameter b we use the method of least squares where the minimization is done by numerical methods. For every model we have calculated the A/E statistics which gives us and information for the fit of the model to the original values, taken from statistical data. On fig. 3 is presented graphical representation of mortality data (survival probability) for 1966 and 2009, and the appropriate Azbel model for the same year. The values of the A/E statistics is 100.1227 for the 1966, and A/E=100.0757 for the mortality table in 2009.

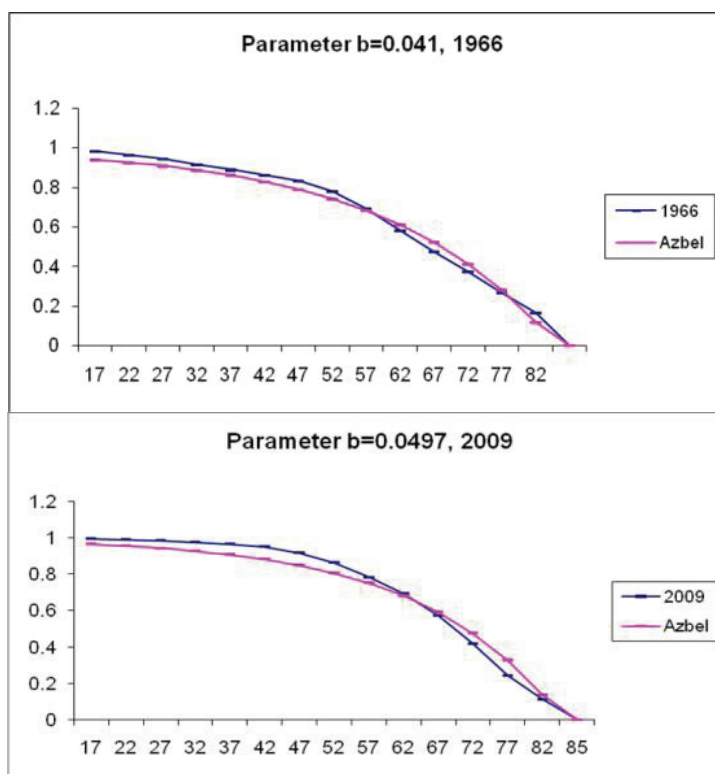


Fig. 3. Graphic presentation of mortality data for the years 1966 and 2009, and data from Azbel model for the same years.

If we assume that the increasing of the parameter  $b$  should be in linear line, it's expected that the value for the mortality table in 1985 to get value of 0.04765, but the real value is 0.04567.

From the values of the parameter  $b$ , we can create the time series, and according to these values we can make forecasting of future values of the parameter  $b$ . On fig. 4 is presented the approximation of the values of parameter  $b$  by linear regression line. The fit of the regression line is acceptable, so we can use it as valid approximation for forecasting future mortality tables.

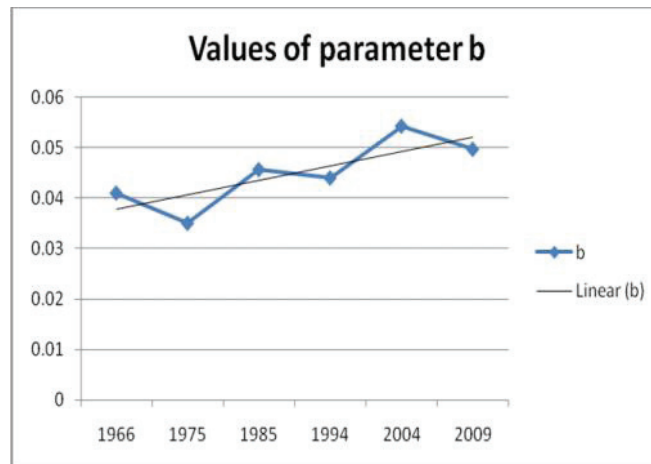


Fig. 4. Approximation of the values of parameter  $b$  by linear regression line

Forecasting of future values of mortality tables is straight forward if we know the approximate value of the parameter  $b$ .

#### 4. Forecasting mortality by time series analysis

For identification and forecasting future mortality values, we should create time series for every different age in the table of mortality for 1966 and 2009. On fig. 2 are presented two time series for different ages. For every different time series we should create ARIMA model, and at the end we can make forecasting few future values. At the end of the time series analysis we get several future mortality tables. By using those mortality tables we can compare their values with the values taken from the Azbel model.

As an example in this paper we have identified the time series for survival probability for the ages 15-19. Those data are presented on fig. 5.

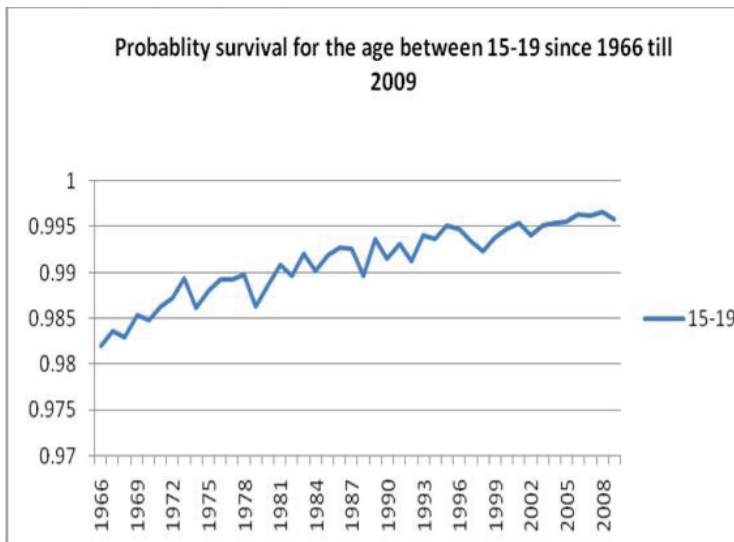


Fig. 5. Data for identification of survival probability

The analyzed time series is non stationary. It has a positive trend.

We need to make differencing on the original data of the series to get the stationary one. From the results of unit root test ADF, we can make conclusion that the differenced series is stationary one. Results are given in table 1.

Table1. Unit root test for the differenced series

ADF Test Statistic	-5.9745	1% Critical Value*	-3.6228
		5% Critical Value	-2.9446
		10% Critical Value	-2.6105

\*MacKinnon critical values for rejection of hypothesis of a unit root.

On the other side, some statistical tests are valid for “rich enough” input in order to get valid results. In the time series we have only 38 data and we should make an interpolation in order to increase the number of the data in the series.

For determination of the model of identification, we use autocorrelation and partial autocorrelation statistics. Results of these statistics and appropriate Q-statistics are given in table 2. From the information given in table 2 we can conclude that most significant is the first lag of the series.

Table 2. Results of ACF and PACF statistics

Sample: 1966 2009  
Included observations: 44

Auto correlation	Partial Correlation	AC	PAC	Q-Stat	Prob	
*** .	*** .	1	-0.553	-0.553	12.566	0.000
.  *.	.  *.	2	0.191	-0.165	14.108	0.001
. ** .	. ** .	3	-0.244	-0.314	16.689	0.001
.  *.	.  *.	4	0.194	-0.131	18.365	0.001
.  .	.  .	5	-0.047	-0.001	18.467	0.002
.  *.	.  **.	6	0.157	0.215	19.641	0.003
*** .	.  *.	7	-0.323	-0.149	24.752	0.001
.  **.	.  *.	8	0.199	-0.086	26.753	0.001
.  *.	.  .	9	-0.063	0.012	26.959	0.001
.  *.	.  .	10	0.110	0.015	27.615	0.002

. *   .	.   * .	11	-0.059	0.082	27.813	0.003
.   .	.   .	12	-0.039	0.014	27.901	0.006
.   .	.   .	13	-0.016	0.001	27.916	0.009
.   **	.   **	14	0.219	0.230	30.945	0.006
. **   .	.   .	15	-0.238	-0.022	34.703	0.003
.   * .	.   .	16	0.102	-0.054	35.422	0.003

In table 3, is given the model for the analyzed time series with all associated statistics. The model contains constant (interceptor) and one MA(1) parameter which makes it the IMA model of identification. The T-statistics of the model has significant value which indicates that MA(1) is valid for given model. The other statistics like Durbin-Watson, Schwarz criteria and Akaike criteria shows that there isn't a correlation between the residuals of the model. The residuals can be considered as white noise series.

Table 3. MA parameter and associated statistics (E-Views)

Dependent Variable: ALFA

Method: Least Squares

Date: 08/16/11 Time: 09:22

Sample: 1966 2008

Included observations: 43

Convergence achieved after 12 iterations

Backcast: 1965

Variable	Coefficient	Std. Error	t-Star.	Prob.
C	0.000305	4.50E-05	6.778225	0.0000
MA(1)	-0.840780	0.074635	-11.26520	0.0000
R-squared	0.406731	Mean dependent var		0.000354
Adjusted R-squared	0.390252	S.D. dependent var		0.001807
S.E. of regression	0.001411	Akaike info criterion		-10.2382
Sum squared resid	7.16E-05	Schwarz criterion		-10.1520
Log likelihood	196.5270	F-statistic		24.68078
Durbin-Watson stat	1.946164	Prob(F-statistic)		0.000017
Inverted MA Roots	.84			

The same procedure of identification and forecasting should be made for all distinct age values. At the end results of the both models can be compared, and can be used to make decisions for the values of future mortality. As final result we can take the results for the best fit model, or the average value of both models.

However, the time series analysis for mortality table is a less efficient and cumbersome approach towards modeling and it gives several future values of mortality tables. At the end the forecasted values from both models are very close. According to this, as simplest and at the same time very efficient model (if this criteria should be used as crucial) we can choose an Azbel model as a favorite for modeling mortality tables and functions. By using this model we are not limited to forecast only several future values which is very important in actuary for long time projection of mortality, but we should test the forecasting every year with new gained statistical data.

#### 4. Conclusions

Changes in the mortality tables are important for calculation of annuity and insurance premium. The real expectation of technical reserves is also connected with the changes in mortality tables. If we make good forecast of the values in mortality tables we can make appropriate calculations of annuity, insurance premium and technical reserves.

In this paper we have made identification on mortality tables by using two different models of identification. The Azbel model is simple and efficient model at the same time. The Azbel model is suitable for modeling mortality functions and tables because the future values of mortality are given as function parameter. Hence, we can make forecast for long period, which is very important in the field of life insurance. The accuracy of identification is not very high, but that cannot be expected for the long period of forecast.

Identification of future mortality using time series analysis is not straight forward as it is with Azbel model. For every distinct age value, we should create a time series. For all this time series we should make identification of the series and forecasting of several future values. It is great amount of work if we know that we have at list 75 different time series for the whole set of ages. The valid forecasting can be calculated for few year in the future. That is not enough for the need of actuary analysis of premium, technical reserves and other financial parameters. So, we can conclude that the Azbel model is better alternative for identification and forecasting mortality data.

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