ON THE DESIGN OF IIR DIGITAL FILTERS

Mitko Kostov[1], Momcilo Bogdanov[2]

[1] St. Kliment Ohridski University, Faculty for Technical Sciences, Bitola, R. Macedonia, e-mail: mitko.kostov@uklo.edu.mk
[2] SS Cyril and Methodius University, Faculty of Electrical Engineering, Skopje, R. Macedonia, e-mail: bogdanov@cerera.etf.ukim.edu.mk

Abstract: This paper considers the constraints imposed on the values of the design parameters in the efficient method proposed by Hedge and Shenoi. We have examined the relationships between the filter parameters in order to make correct specifications of the permissible values of the design parameters.

Index terms: Analytical design method, IIR digital filters, magnitude and group delay approximation.

1. INTRODUCTION

It is well known that infinite-impulse response (IIR) filters can be designed by transforming an analog prototype filter to the digital domain using either impulse invariant or bilinear transformations. These transformations guarantee the stability of the filter and preserve, to a certain extent, its magnitude response. There are also methods to design IIR filters directly in the digital domain. However, all these methods, analog to digital or strictly digital, consist in approximating only the magnitude response. On the other hand, in certain applications such as pulse transmission and high-speed data transmission, it is desirable the filters have constant delay characteristics.

In [1], R. Hedge and B. Shenoi present a new technique to approximate the magnitude response of IIR filters with maximally flat or equiripple delay characteristics. The magnitude response can be matched to prescribed values at a number of frequencies between $\omega = 0$ and $\omega = \pi$ and has desired degrees of flatness at $\omega = 0$ and/or $\omega = \pi$.

Motivated by this efficient method, we have examined the relationships between the filter parameters in order to make correct specifications of the permissible values of the design parameters. To obtain an IIR filter that has to meet magnitude and phase specifications simultaneously, the design parameters have to satisfy certain constraints. In [1] the set of all filter parameters is not equal to the set of design parameters, i.e. there are relationships between the filter parameters, so they can not be all specified at the beginning of the design process. Our paper considers the constraints imposed on the values of the design parameters in Hedge and Shenoi method. It is organized as follows. Section II gives an outline of the Hedge and Shenoi method. Section III presents the relationship between the design parameters. A summary is provided in section IV.

2. OUTLINE OF THE HEGDE AND SHENOI METHOD

This section gives a review of the Hedge and Shenoi method [1]. In [1], using two theorems, they derive sufficient conditions to obtain desired degrees of flatness at any given point in the frequency domain. From these conditions, which are linear in terms of the coefficients of the numerator polynomial, they obtain an analytical solution to the problem of designing IIR digital filters approximating constant passband delay characteristics (in either maximally flat or equiripple sense) and flat passband and stopband magnitude characteristics. Their method also meets the prescribed magnitudes at a number of frequencies, in addition to the zero frequency and half Nyquist frequencies.

First, in [1] an all-pole transfer function $H(z)=1/D(z)$ is chosen. The denominator polynomial of this function is chosen such that the filter has a maximally flat delay characteristic [2], i.e.

$$D(z)=\frac{P!}{(2P)!}\prod_{i=P+1}^{2P}(2\tau+i)\left\{\sum_{k=0}^{P}\left[(-1)^k\binom{P}{k}\cdot\prod_{i=0}^{P}\left(\frac{2\tau+i}{2\tau+k+i}\right)\right]z^{-k}\right\}$$

where τ is the group delay of $1/D(z)$ at the origin and P is the order of $D(z)$.

This function is then augmented by a mirror image polynomial as its numerator so that the delay characteristic of the resulting transfer function remains unaltered except by a pure delay, i.e.

$$H(z)=\frac{N(z)}{D(z)}$$

where

$$N(z) = z^{-p} N_a(z), \quad N_a(z)=b_0+b_1\left(\frac{z+z^{-1}}{2}\right)+\cdots+b_p\left(\frac{z^p+z^{-p}}{2}\right).$$

The coefficients of $N_a(z)$ denoted by $b_0, b_1, \ldots, b_p$ are chosen so that the function $H(z)$ has the desired magnitude characteristic.

The desired conditions 1) degree of flatness L at $\omega=0$, 2) degree of flatness K at $\omega=\pi$, 3) magnitudes of 1 and 0 at $\omega=0$ and $\omega=\pi$, respectively, and 4) a 3dB attenuation at the specified bandwidth ω_b can be expressed jointly in matrix form as

$$\mathbf{A_1 b_1} = \mathbf{d_{1I}},$$

where

$$\mathbf{b_1} = [b_0\ b_1\ b_2\ \ldots\ \ldots\ \ldots\ b_p]^{\mathrm{T}},$$

$$\mathbf{d_{1I}} = [D^{(0)}\ 0\ D_{\omega_b}\ D^{(2)}\ D^{(4)}\ \ldots\ D^{(M)}\ 0\ \ldots\ 0]^{\mathrm{T}},$$

$$D_{\omega_b} = 0.7071\ |D(e^{j\omega_b})|,$$

$$D^{(k)} = \frac{d^k\left|D(e^{j\omega})\right|}{d\omega^k}\bigg|_{\omega=0,\pi} = 0,$$

$$M = L-1, \quad N = K-1, \quad p = (L+K+2)/2.$$

$$
\boldsymbol{A_1} = \begin{bmatrix}
1 & 1 & 1 & \cdots & 1 \\
1 & (-1)^1 & (-1)^2 & \cdots & (-1)^p \\
1 & \cos\omega_b & \cos 2\omega_b & \cdots & \cos p\omega_b \\
0 & -(1^2) & -(2^2) & \cdots & -(p^2) \\
0 & (1^4) & (2^4) & \cdots & (p^4) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & (-1)^{M/2} & (-1)^{M/2}(2^M) & \cdots & (-1)^{M/2}(p^M) \\
0 & 1 & -(2^2) & \cdots & -(-1)^p p^2 \\
0 & -1 & (2^4) & \cdots & (-1)^p p^4 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & -(-1)^{N/2} & -(-1)^{N/2}(2^N) & \cdots & (-1)^p(-1)^{N/2} p^N
\end{bmatrix}. \quad (1)
$$

The analytical solution to the problem of designing IIR filters with maximally flat delay characteristics and a magnitude response that has desired degrees of flatness at $\omega = 0$ and $\omega = \pi$ is given by $\mathbf{b_1} = \mathbf{A_1^{-1}}\,\mathbf{d_{1I}}$. With this method it is possible to specify the magnitude of the low-pass filter at multiple frequencies in the range [0, π]. This simply adds more linear equations in (1).

3. CONSTRAINTS

To obtain an IIR filter that has to meet magnitude and phase specifications simultaneously, its design parameters have to satisfy certain constraints. This section presents the relationships between the design parameters in the Hedge and Shenoi method [1]: the stopband frequency ω_b, the group delay τ, the order P of the transfer function denominator and the degrees of flatness L and K.

First, in order to obtain a magnitude response with degrees of flatness L at $\omega = 0$ and K at $\omega = \pi$, the values of the parameters τ and P must be chosen such that the delay characteristic of the function $1/D(z)$ approximates the constant group delay τ in the maximally flat sense in the region $0 \le \omega \le \omega_b$. If the parameter P increases, while the parameters τ and ω_b do not change, the region where the delay characteristic of the function $1/D(z)$ approximates the parameter τ, increases as well. This is shown in Figure 1 where solutions for the group delay are obtained for the following filter parameters ω_b=0.35π, τ=0.5, L=3, K=3. Then, if the group delay τ increases, while the parameters P and ω_b do not change, the region where the delay characteristic of the function $1/D(z)$ approximates the constant group delay τ decreases. This is shown in Figure 2 where solutions for the group delays are obtained for the following filter parameters ω_b=0.35π, P=6, L=3 and K=3.

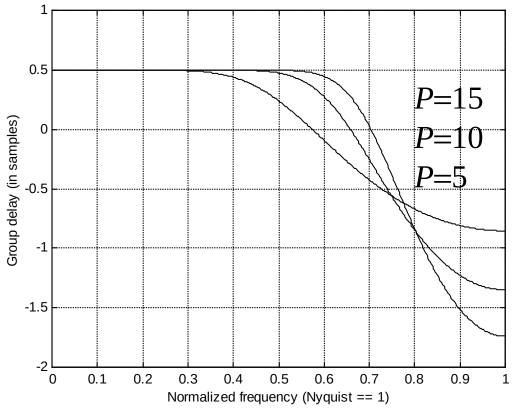


Figure 1. Group delay of maximally flat delay filter with desired degrees of flatness at both $\omega = 0$ and $\omega = \pi$

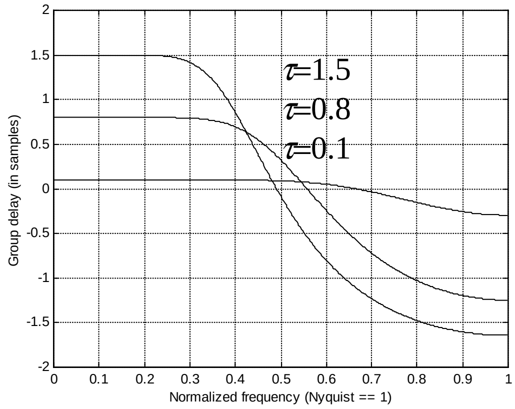


Figure 2. Group delay of maximally flat delay filter with desired degrees of flatness at both $\omega = 0$ and $\omega = \pi$

The parameters P, τ and ω_b, affect the magnitude characteristic of the filter $H(z)$ as follows. Increasing the value of P improves the passband of the filter, but worseens its stopband, as shown in Figure 3. The filter parameters in Figure 3 are ω_b=0.2π, τ=1, L=5 and K=13. On the other hand, decreasing the value of τ improves the passband of $H(z)$, but worsens its stopband (Figure 4). The filter parameters in Figure 4 are ω_b=0.35π, P=6, L=9 and K=5.

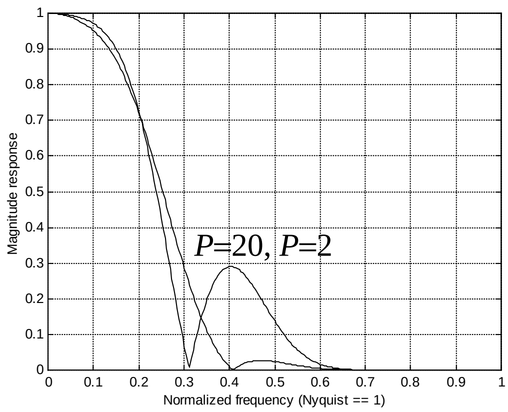


Figure 3. Magnitude response of maximally flat delay filter with desired degrees of flatness at both $\omega = 0$ and $\omega = \pi$

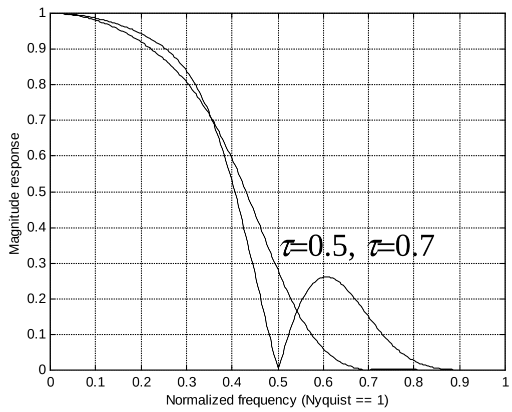
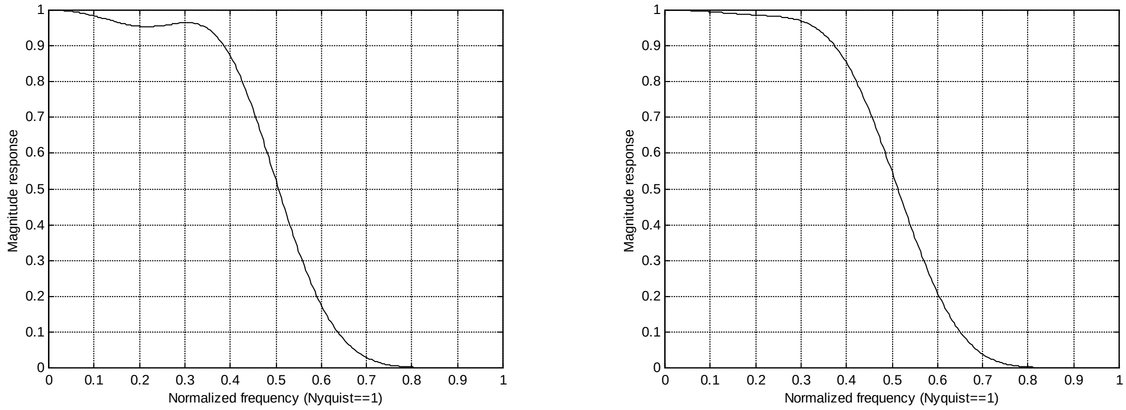


Figure 4. Magnitude response of maximally flat delay filter with desired degrees of flatness at both $\omega = 0$ and $\omega = \pi$

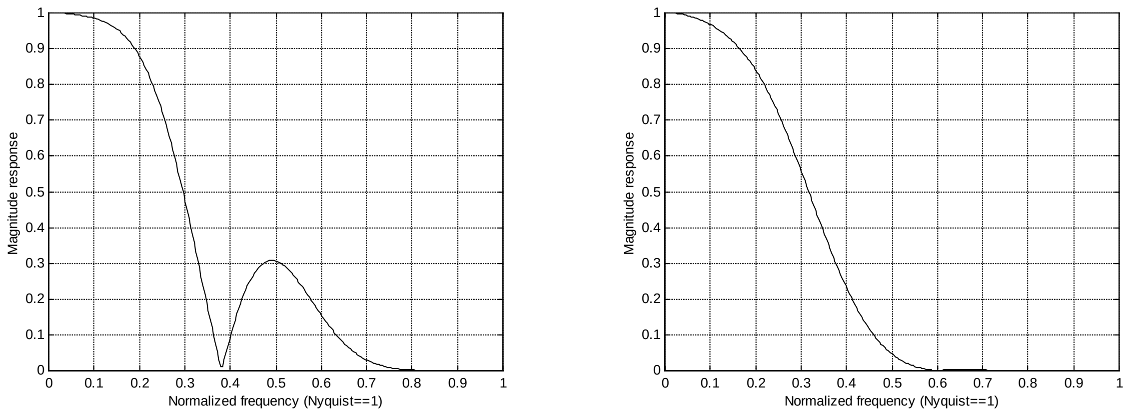
Then, if the filter designed with [1] has a relatively high frequency ω_b, its magnitude has poor passband, but good stopband. Therefore, we recommend choosing a small value for the parameter τ or a big value for the parameter P (Figure5). For filter parameters ω_b=0.45π, τ=0.6, P=5, L=5 and K=7, there is a ripple in the magnitude response (shown in Figure5-a). The ripple can be eliminated by choosing τ=0.2 and P=15 (shown in Figure5-b).



a) b)

Figure 5. Magnitude response of maximally flat delay filter with desired degrees of flatness at both $\omega = 0$ and $\omega = \pi$ for filter parameters

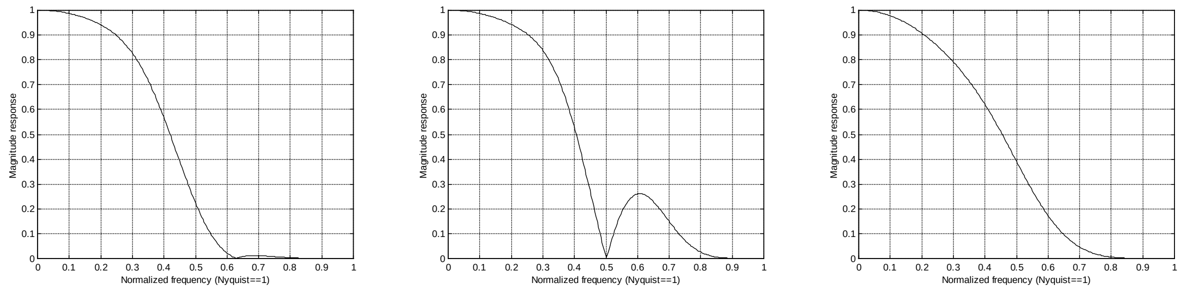
If the filter has a relatively low stopband frequency ω_b, its magnitude has poor stopband, but good passband. So, if it is important the filter to have a good stopband, then we recommend choosing a big value for the parameter τ or a small value for the parameter P (Figure6). For filter parameters ω_b=0.25π, τ=0.5, P=6, L=5 and K=7, there is a ripple in the magnitude response (shown in Figure6-a). The ripple can be eliminated by choosing τ=0.9 and P=3 (shown in Figure6-b).



a) b)

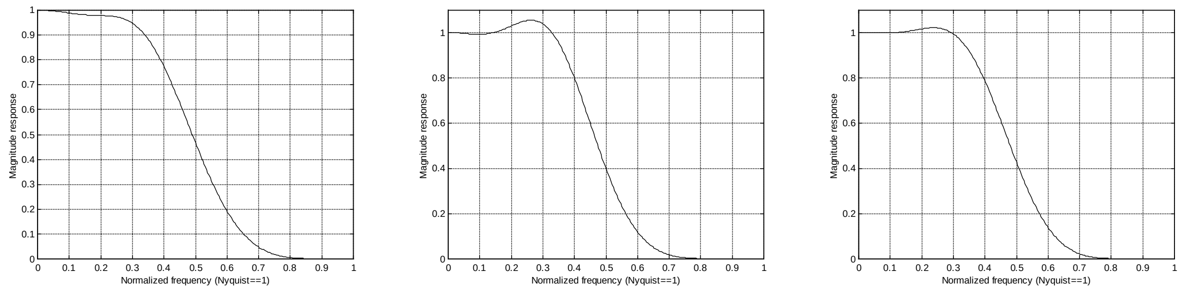
Figure 6. Magnitude response of maximally flat delay filter with desired degrees of flatness at both $\omega = 0$ and $\omega = \pi$ for filter parameters

Finally, the degrees of flatness at $\omega = 0$, L, and $\omega = \pi$, K, cannot be chosen completely freely, as they have to obtain adjacent values. If the degrees of flatness are very different, there will be distortion in the magnitude characteristic of the filter. In addition, if the parameter L is much greater then the parameter K, there will be distortion in the stopband of the magnitude characteristic. In such a case, big values of the stopband frequency ω_b and the parameter τ or a small of the parameter P would improve the stopband. For instance, for L=7 and K=5 (τ=0.5, ω_b=0.35π, P=6), the magnitude characteristic of the filter is quite good (shown in Figure7-a), but for L=9 and K=5, there is a ripple in the stopband (shown in Figure7-b), which can be eliminated by choosing τ=0.8 with the other parameters unchanged (Figure7-c). On other hand, if the parameter L is much smaller then the parameter K, there will be distortion in the passband of the magnitude characteristic. In such a case, small values of the stopband frequency ω_b and the parameter τ or a big value of the parameter P should be used. For instance, for L=3 and K=5 (τ=0.5, ω_b=0.42π, P=3), the magnitude characteristic of the filter is quite good (shown in Figure8-a), but for L=3 and K=7, there is a ripple in the passband (shown in Figure8-b), which can be eliminated by choosing τ=0.2 and P=20 with the other parameters stay unchanged (Figure8-c).



a) b) c)

Figure 7. Magnitude response of maximally flat delay filter with desired degrees of flatness at both $\omega = 0$ and $\omega = \pi$



a) b) c)

Figure 8. Magnitude response of maximally flat delay filter with desired degrees of flatness at both $\omega = 0$ and $\omega = \pi$

4. SUMMARY

We considered the constraints imposed on the values of the design parameters in Hedge and Shenoi method. We showed how the choice of the values of the stopband frequency ω_b, the group delay τ and the order of the denominator polynomial P, affect the magnitude and the delay characteristics. Increasing P and decreasing τ yields an increase in the region where the delay characteristic of the function $1/D(z)$ approximate the group delay. Increasing P and decreasing ω_b and τ improves the passband of the filter, but worsens its stopband. Finally, the degrees of flatness, L and K, must obtain adjacent values. Otherwise, there will be distortion in the magnitude of the filter, which may be eliminated by choosing appropriate values for the other design parameters.

REFERENCES

[1] R. Hedge and B. A. Shenoi, "Magnitude Approximation of Digital Filters with Specified Degrees of Flatness and Constant Group Delay Characteristics," *IEEE Trans. Circuits Syst. II*, vol. 45, pp. 1476-1486, Nov. 1998.

[2] J. P. Thiran, “Recursive Digital Filters with Maximally Flat Group Delay,” *IEEE Trans. Circuit Theory*, vol. CT-18, pp. 659-664, Nov. 1971.