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Design of QMF Banks with Reduced Number of Iterations

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Abstract

The iterative algorithme of Chen and Lee for the design of quasiequiripple QMF banks in the frequency domain is modified; the weighting function incorporated in the objective function is updated at the end of each iteration. As a result, the number of iteration and the number of floating point operations are reduced significantly.

I. Introduction

Quadrature mirror filter (QMF) banks find applications in many areas such as image compression, subband coding for speech processing, and transmultiplexers for telecomunications. Many methods for the design of linear-phase two-channel filter banks have been reported in the literature since the mid-1970's. It is known that the design of QMF banks in the frequency domain can be accomplished by using least-squares and minimax methods. On the other hand, it has been shown that the minimax design can be performed if an adequately updated weighting function is included in a least-squares objective function [1]–[3]. The design is a typical unconstrained and highly nonlinear optimization problem due to the fact that the objective function is a fourth-order function of the design parameters. In [1] Chen and Lee have proposed a linearization technique and derived an analytical design formula. Based on this formula, the coefficients of the required low-pass filter can be obtained by solving a set of linear equations at each iteration. They also have incorporated the proposed technique with a weighted least-squares algorithm [2]. Thus they have obtained QMF banks having overall reconstruction error minimized in the minimax sense, in addition to the QMF filters having least-squares stopband error.

The method of Chen and Lee leads to a very efficient algorithm for the considered design problem. An improved implementation of this algorithm, concerning the evaluation of two integrals involved in the computation of the objective function, has been reported in [4]. A recently developed a new iterative method [5] is also based on the same algorithm, and the improvements result from the formulation of perfect reconstruction condition in the time domain.

In this paper we propose another way of using a weighted function to minimize the overall reconstruction error in the minimax sense. It consists in updating the weighted function at the end of each iteration. As a result, the number of iterations is significantly reduced, while the QMF banks have practically the same characteristics as those obtained by the original algorithm. The number of floating point operations is also reduced.

The paper is organized as follows. Section II briefly describes the original algorithm due to Chen and Lee [1]. Section III introduces our proposal for an alternative realization of the algorithm, along with two design examples for illustration and comparison. The conclusions are given in Section III.

II. The Basic Idea and Structure of the Original Algorithm

Chen and Lee use the following notation for a two-channel QMF bank: H_0 and H_1 are the lowpass and highpass filters, respectively, of the analysis section, and F_0, F_1 are the corresponding filters of the synthesis section. Their impulse responses are $h_0(n)$, $h_1(n)$, $f_0(n)$ and $f_1(n)$. All filters are assumed to be linear phase and they are all of length N, where N is even. The frequency response of the lowpass analysis filter, $H_0(e^{j\omega})$, satisfies power complementary property

$$T(e^{j\omega}) = |H_0(e^{j\omega})|^2 + |H_0(e^{j(\omega+\pi)})|^2 \text{ for all } \omega .$$

The reconstruction error is defined as $e_r(\omega) = T(e^{j\omega}) - 1$, and the objective function E to be minimized in the weighted least-squares sense is given by

$$\begin{aligned} E &= E_r + E_s \\ &= \int_0^{\pi} W(\omega) e_r^2(\omega) d\omega + \alpha \int_{\omega_s}^{\pi} |H_0(e^{j\omega})|^2 \, d\omega \end{aligned}$$

where E_r is the energy of the reconstruction error, E_s is stopband energy related to H_0, α is the relative weight between E_r and E_s, $W(\omega)$ is a weighting function, and ω_s is the stopband frequency.

A basic idea in the algorithm [1] is that the conditions for a perfect reconstruction system are satisfied when the frequency response of the lowpass synthesis filter $F_0(e^{j\omega})$ is very close to the frequency response of the lowpass analysis filter H_0 at kth iteration $H_0^{(k)}(e^{j\omega})$. Chen and Lee have reformulated the design problem as follows. The coefficients $\{f_0(n), n = 0, 1, ..., N/2-1\}$ of the lowpass synthesis filter F_0 must be found such that the overall error function

$$E = \int_0^{\pi} W(\omega)[T(e^{j\omega}) - 1]^2 + \alpha \int_{\omega_s}^{\pi} |F_0(e^{j\omega})|^2 \, d\omega \tag{1}$$

is minimized at the $(k+1)$th iteration, where $T(e^{j\omega})$ can be expressed as

$$\begin{aligned} T(e^{j\omega}) &= H_0^{(k)}(e^{j\omega}) F_0(e^{j\omega}) \\ &\quad + H_0^{(k)}(e^{j(\omega+\pi)}) F_0(e^{j(\omega+\pi)}). \end{aligned}$$

The frequency responses of the QMF filters and the related error functions are evaluated on a dense grid of frequency

$$\Omega = \{\omega_1, \omega_2, \cdots, \omega_k = \omega_s, \cdots, \omega_L\}$$

linearly distributed in the range from $\omega = 0$ to $\omega = \pi$. The matrix form of the objective function E at kth iteration is given by

$$\begin{aligned} E^{(k)} &= [UQ - I]^T W [UQ - UI] \\ &\quad + \alpha [U_s Q]^T [U_s Q]. \end{aligned} \tag{2}$$

For the matrices included in (2) and other details we refer to the original paper [1]. Here we list only the expressions mentioned in the summary of Chen and Lee's algorithm.

The weighting matrix W associated with the weighting function $W(\omega)$ is

$$W = \text{diag}[W(\omega_1), \cdots W(\omega_s), \cdots W(\omega_L)]. \tag{3}$$

The coefficient vector

$$Q = [\, f_0^{(k)}(0), f_0^{(k)}(1), \cdots f_0^{(k)}(N/2-1)]^T$$

is obtained by solving the following set of linear equations

$$Q = [U^T W U + \alpha U_s^T U]^{-1} U^T W I . \tag{4}$$

The coefficients of the lowpass analysis filter H_0 at the $(k+1)$th iteration are computed by the updated formula

$$h_0^{(k+1)}(n) = (1-\tau) h_0^{(k)}(n) + \tau f_0^{(k+1)}(n) \tag{5}$$

for $0 \le n \le N/2-1$ and $0 < \tau < 1$.

The design process is terminated when

$$\frac{\left| E^{(k+1)} - E^{(k)} \right|}{E^{(k+1)}} < \varepsilon . \tag{6}$$

The weighting function $W(\omega)$ used in (3) and (4) is adjusted appropriately from iteration to iteration in order to minimize the reconstruction error in the minimax sense. At the $(k+1)$th iteration

$$W^{(k+1)}(\omega) = W^{(k)}(\omega) v^{(k)}(\omega) \tag{7}$$

where $v^{(k)}(\omega)>0$ is the required update at the kth iteration and $v^{(k)}(\omega_i)> v^{(k)}(\omega_j)$ if

$$|e_r^{(k)}(\omega_i)| > |e_r^{(k)}(\omega_j)|. \qquad (8)$$

The function $v^{(k)}(\omega)$ is obtained by using an envelope function $B^{(k)}(\omega)$ whose construction is explained in [1]

$$B^{(k)}(\omega) = \frac{\omega - \omega_J}{\omega_{J+1} - \omega_J} V^{(k)}(J+1) + \frac{\omega_{J+1} - \omega}{\omega_{J+1} - \omega_J} V^{(k)}(J). \qquad (9)$$

Finally

$$v^{(k)}(\omega) = \frac{L[B^{(k)}(\omega)]^\theta}{\sum_{i=1}^{L} W^{(k)}(\omega_i)[B^{(k)}(\omega_i)]^\theta}. \qquad (10)$$

The parameter θ affects the convergence ($\theta = 1.5$ is recommended as an optimal choice). The design process is terminated when the resulting reconstruction error is "equiripple" enough. A stopping criterion is

$$\frac{\max(V) - \min(V)}{\max(V)} \leq \kappa \qquad (11)$$

where $\max(V)$ and $\min(V)$ denote the maximum and the minimum values of the reconstruction error over all extremal frequencies, respectively, and κ is a positive constant.

The flowgraph of the algorithm due to Chen and Lee is shown in Fig. 1.

III. Alternative Realization of the Algorithm

The algorithm of Chen and Lee includes a minimization of the objective function (1) in the least square sense. The weighted function $W(\omega)$ which is incorporated in (1) stays invariant during this minimization. Only when the relative error at the $(k+1)$ th iteration (6) is smaller than ε is the minimization performed using the minimax criterion. However, the weighted function depends on the reconstruction error $e_r(\omega)$, which changes from iteration to iteration. Taking this fact into account we can expect that the number of iterations would be reduced if $W(\omega)$ is updated at the end of each iteration. A reduced number of iterations results in a decrease in the total number of operations. Our computer simulations show that the decrease in the number of operations is significant. The modified version of Chen and Lee's algorithm is depicted in Fig. 2.

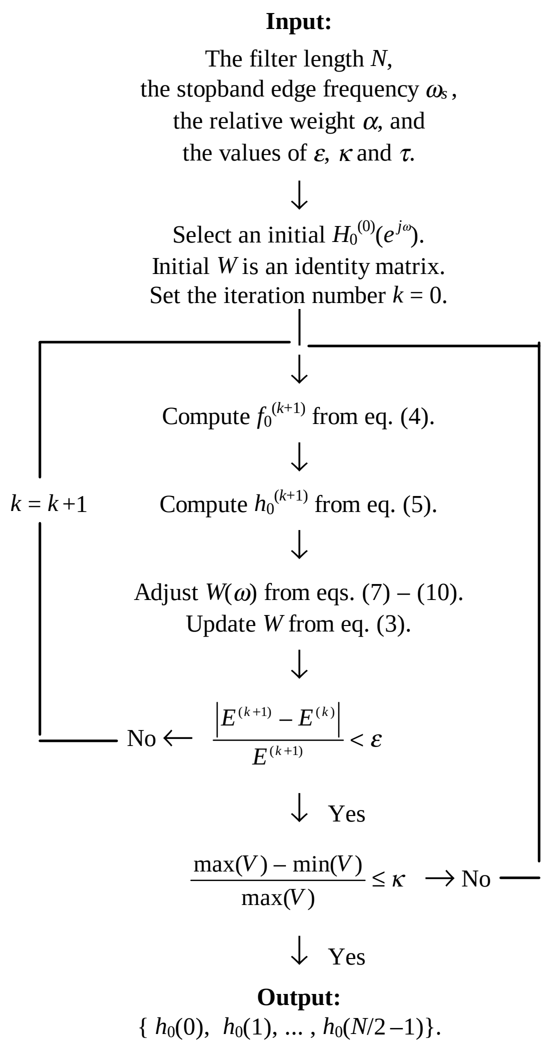

Fig. 1. Flowgraph of the original algorithm.

The modified algorithm also differs from its original by its output coefficients. As it can be seen from Fig. 2, our algorithm outputs the coefficients $\{f_0(n)\}$, while Chen and Lee's algorithm outputs $\{h_0(n)\}$. It follows from (5) that when the relation (6) is satisfied, $\{f_0(n)\}$ is different from $\{h_0(n)\}$. On the other hand, $\{f_0(n)\}$ is the unique vector which minimizes the objective function (1). Hence, setting $\{h_0(n)\} \leftarrow \{f_0(n)\}$ instead of $\{f_0(n)\} \leftarrow \{h_0(n)\}$ produces filters with somewhat better frequency responses.

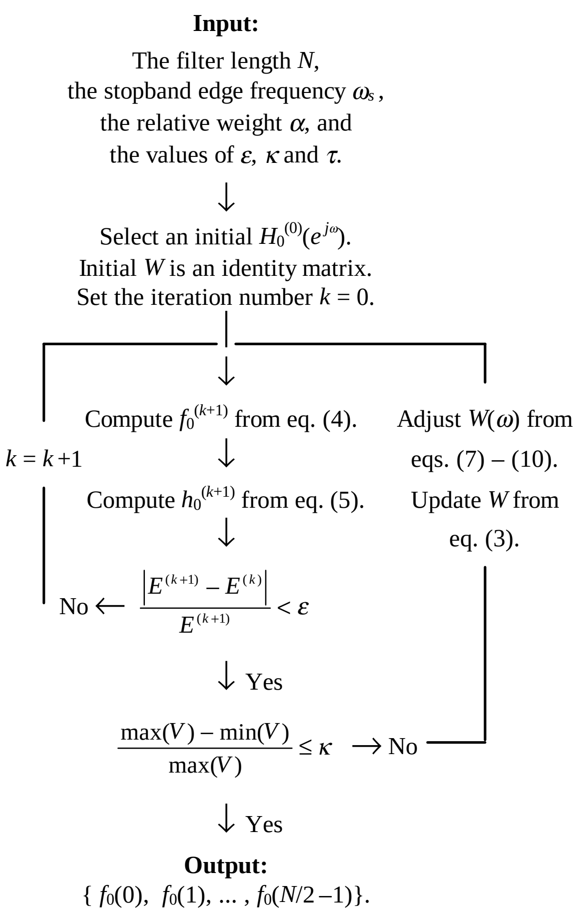

Fig. 2. Flowgraph of the modified algorithm.

For illustration and comparison we include two examples with the same design parameters and the same initial filters for starting the iterative process as in [1].

Example 1: This example is the same as example 3 in [1]. The design parameters are: $N = 32$, $\omega_s = 0.6\pi$, $\alpha = 1$, $\varepsilon = 0.001$, $\kappa = 0.02$, $\tau = 0.5$, and $\theta = 1.5$. The coefficients of the initial filter are $h_0^{(0)}(n) = 0.5$ for $n = N/2-1$, $N/2$, and $h_0^{(0)}(n) = 0$, elsewhere. The signals in the frequency domain are discretized to $L = 8N = 256$ samples. Table 1 lists the stopband edge attenuation of the lowpass analysis filter, the peak reconstruction error of the resulting QMF bank, the number of iterations and the number of floating point operations in millions (Mflops).

TABLE 1

Algorithm	original	modified
Stopband atten.	36.268 dB	36.336 dB
Peak rec. error	0.0124 dB	0.0123 dB
Number of iter.	20	12
Mflops	178. 539	107. 203

Example 2: The design parameters are the same as those used in the previous example. A better initial filter for starting the iterative process is generated by the Remez exchange algorithm. This filter satisfy the following specifications:

$$H_0^{(0)}(e^{j\omega}) = \begin{cases} 1 & \text{for } 0 \le \omega \le \pi - \omega_s \\ 1/\sqrt{2} & \text{for } \omega = 0.5 \\ 0 & \text{for } \omega_s \le \omega \le \pi. \end{cases}$$

The weighting function is

$$W(\omega) = \begin{cases} 1 & \text{for } 0 \le \omega \le \pi - \omega_s \\ \sqrt{2} & \text{for } \omega = 0.5 \\ 0 & \text{for } \omega_s \le \omega \le \pi. \end{cases}$$

The results are shown in Table 2.

TABLE 2

Algorithm	original	modified
Stopband atten.	36.400 dB	36.336 dB
Peak rec. error	0.0124 dB	0.0124 dB
Number of iter.	15	10
Mflops	133. 944	89. 349

III. SUMMARY

We have considered Chen and Lee's algorithm for an iterative design of QMF banks in the frequency domain. We proposed an alternative realization of this algorithm which differs from the original in two details. First, the weighted function incorporated in the objective function is updated at the end of each iteration, thus adequately following changes of the reconstruction error function. As a result, the QMF banks are obtained in fewer iterations and a smaller number of floating point operations. Our computer simulations show that the improvement in the number of operations is at least 40%. Second, the set of output coefficients is somewhat different from those in the original algorithm. This yields to a slight improvement in the filters' frequency responses.

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