Mathematical Modeling and Simulation of Hybrid Electric Vehicle

Gordana Janevska [1], Mitko Kostov[1] and Goran Stojanovski[1]

Abstract – **The paper presents a Hybrid Electrical Vehicle modeling based on multi-physics approach. The integrated simulation model of a series-parallel HEV is developed including the electric drive system, PI controller, vehicle load model, and gear box. The system step response is simulated and analyzed. The simulation results confirm the validity of the model.**

Keywords – **Mathematical modeling, Simulation, Hybrid electric vehicles.**

I. Introduction

Hybrid Electrical Vehicles (HEVs) are vehicles with many electric components compared to conventional ones. In fact, the power train consists of electrical machines, power electronics and electric energy storage system connected to mechanical components and to an Internal Combustion Engine (ICE). The approach for a new vehicle design has to be multidisciplinary in order to take into account the dynamic interaction among all the components of the vehicle and the power train itself. In order to find the correct size of the components, the best energy control strategy and to minimize the vehicle energy consumption the vehicle designers need modeling and simulation since prototyping and testing are expensive and complex operations. Developing a simulation model with a sufficient level of accuracy for all the different components based on different physic domains is a challenge.

A HEV uses both an ICE and an electric motor/generator for propulsion. According to the architecture of hybrid propulsion, there are three basic layouts of HEVs:

- **series hybrid** in which the ICE and the electric motor are connected in series, and only the electric motor is providing mechanical power to the wheels;
- **parallel hybrid** in which the ICE and the electric motor are connected in parallel, and both the electric motor and the ICE can deliver mechanical power to the wheels;
- **Power-split hybrid (series–parallel HEV)** which combines both parallel and series hybrid architectures. This architecture is more efficient overall, but at the cost of more complicated control systems.

In this paper, the series-parallel hybrid form is studied. The paper is organized as follows: Section II details the components used in the HEV model. This includes the formulations for the electric drive system, vehicle load model, and gear box. Section III describes the development of the HEV model using Matlab/Simulink software package. Section IV provides simulation results, and Section V concludes the paper.

[1]Gordana Janevska, Mitko Kostov and Goran Stojanovski are with the Faculty of Technical Sciences at St. Kliment Ohridski University, Bitola, Makedonska Falanga 37, Bitola 7000, Macedonia,
E-mail: gordana.janevska@uklo.edu.mk

II. Series–Parallel HEV Model Architecture

The series–parallel HEV (Fig. 1) incorporates the features of both series and parallel HEVs. Therefore, it can be operated as a series or parallel HEV. In comparison to a series HEV, the series–parallel HEV adds a mechanical link between the engine and the final drive, so the engine can drive the wheels directly. When compared to a parallel HEV, the series–parallel HEV adds a second electric motor that serves primarily as a generator.

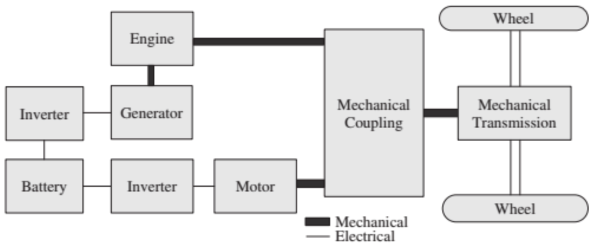

Fig. 1. The architecture of a series–parallel HEV

A. Electric Drive System

The high voltage *battery* is the main power source for the electric motor drive. For the purpose of this modeling, an ideal battery source is assumed, which is modeled by using a simple voltage source with zero source impedance.

The main function of *inverter* is to convert the DC voltage to an AC voltage. The inverter is a major part of the motor drive circuit and it allows controlling the current in the motor, hence the motor produces smooth torque output. In this paper, inverter is modeled as an ideal three-phase current source from the controller.

Main function of an electric drive is to convert electrical energy into mechanical energy. This is accomplished by the use of an electric machine or a motor. Here, as part of the system modeling, a *Permanent Magnet Synchronous Motor* (PMSM) is used.

Motor torque as a function of speed is given by

$$\tau = \tau_s - K\frac{1}{\omega_{NL}}\omega \ , \qquad (1)$$

where τ_s is the stall torque, ω_{NL} is no load rotational speed, and K is a constant. Motor power as a function of rotational speed is

$$P = \tau \cdot \omega \ . \tag{2}$$

A PMSM used in the electric drive can be modeled using the steady state equations. PMSM motor phases is configured as a wye connection.

State equations for the motor phase current are:

$$\frac{di_a}{dt} = \frac{1}{L}(v_a - i_a R_a - v_{e\_a} - v_n) \ , \tag{3}$$

$$\frac{di_b}{dt} = \frac{1}{L}(v_b - i_b R_b - v_{e\_b} - v_n) \ , \tag{4}$$

$$\frac{di_c}{dt} = \frac{1}{L}(v_c - i_c R_c - v_{e\_c} - v_n) \ , \tag{5}$$

where v_a, v_b and v_c are phase drive voltages, R_a, R_b and R_c are motor phase resistances, v_{e_a}, v_{e_b} and v_{e_c} are the phase back-emf voltages, and v_n is the neutral point voltage.

A commonly used technique to achieve the phase alignment and produce the maximal torque is to change the frame of reference from stationary A,B,C frame to rotational D,Q frame of reference. This frame transformation, known as Park's Transformation, converts an AC signal into a DC signal to simplify the analysis and controls.

Since d-axis produces no torque, the measured and controlled d-axis current should be 0. On the other hand, q-axis produces the maximum torque, so the q-axis current should be the actual commanded current.

The block diagram of the model is shown in Fig. 2.

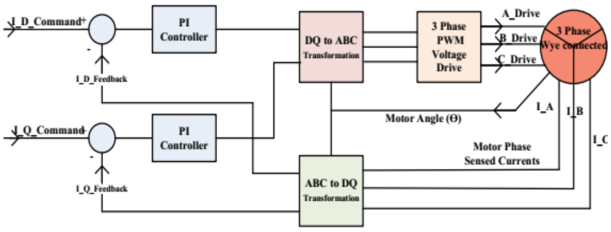

Fig. 2. Closed loop current control

The conversion from stationary frame to the d-q rotating frame leads to the dynamic state equations of the phase currents:

$$\frac{di_d}{dt} = \frac{1}{L_d}(v_d - i_d R_s + \omega_m L_q i_q) \ , \tag{6}$$

$$\frac{di_q}{dt} = \frac{1}{L_q}(v_q - i_q R_s - \omega_m L_d i_d - \omega_m \lambda_m) \ , \tag{7}$$

and the motor torque is given by

$$T_e = \frac{3}{2}\frac{P}{2}\left[\lambda_m i_q + \left(L_d - L_q\right) \cdot i_q i_d\right] , \tag{8}$$

where P is the number of poles in the machine.

B. Vehicle Load Model

Vehicle load can be a function of multiple variables, including aerodynamics. In this study, only the parameters that provide the largest effects on the vehicle in terms of load are considered.

The torque on wheels can be calculated as

$$\begin{aligned} T_{wheel} &= r f_{wheel} = r M_T \frac{dv}{dt} = \\ &= r M_T \frac{d(r\omega_{wheel})}{dt} = r^2 \frac{d(\omega_{wheel})}{dt} \end{aligned} \tag{9}$$

where r is the radius of the wheels, f_{wheel} is the force on the wheels, i.e. accelerating mass of vehicle, M_T is the total mass, M_T=M_{veh}+M_{tire}, v is the velocity of the vehicle, and ω_{wheel} is the rotational speed of the wheel.

The load torque can be calculated as

$$T_L = F_T \cdot r \ , \tag{10}$$

where F_T is the total force and it can be determined by

$$F_T = f_{wheel} + f_{rr} + f_d + f_{slope} + f_{bear} \tag{11}$$

The tire rolling resistance f_{rr} is given by $f_{rr} = \mu_{rr} Mg$, where μ_{rr} is the rolling resistance coefficient. Viscous drag f_d can be calculated as $f_d = (1/2)\rho A C_d v^2$ where ρ is the air density, A is cross section area and C_d is the viscous drag coefficient. Force due to slope effect f_{slope} is given by $f_{slope} = Mg\sin(\theta)$, where θ is the slope angle. Force due to bearing friction f_{bear} is given by $f_{bear} = (k_b/r)\omega$.

The rotational equation of vehicle tire is determined as

$$\frac{d\omega}{dt} = \frac{1}{J}(T_e - T_L) \ , \tag{12}$$

where J is the moment of inertia, and T_e is the motor torque.

C. Gear Box

The problem that occurs in the case of hybrid powertrain configuration is to find the best gear ratios and arrangement that can cover more kilometers with minimal use of electricity from the battery. The problem of minimizing fuel consumption depends on several factors such as gear ratio, engine torque throughout the cycle.

Reducing the gear ratio allows the shaft to spin at a lower rate, and increase the torque, yet get all the power from the motor drive. Since the motor can spin at high rate, the gear ratio allows the motor to be at low torque. This mathematically can be written as

$$P = \tau_m \omega_m = \tau_w \omega_w = \tau_w \frac{\omega_m}{G} \ , \tag{13}$$

where ω_w is the wheel speed, τ_w is the torque at the wheel, and G is the gear ratio.

Speed ratio can be defined as ratio between input engine speed and the output speed of the vehicle. By changing the connection of the planetary gears, the efficiency of the vehicle can be increased [3].

The selection of an appropriate gear ratio can be useful in efficient use of the power of the drive, and minimizing the energy consumption from the power source [4]. Optimizing the gear ratios can be useful for saving energy during travel, resulting in more mileage with one charge of the battery.

III. INTEGRATED SIMULATION MODEL

The system simulation model is developed on the basis of the theoretical mathematical models shown earlier. The five main parts of the overall model are motor model, vehicle model, gear box model, controller model, and model of power balance and energy conservation.

The paper considers the following blocks: the controller, motor model, gear box and vehicle load model. Power-energy model is used to calculate and store the values of input and output power and energy during the drive cycle. The integrated simulation model of the system is shown in Fig. 3.

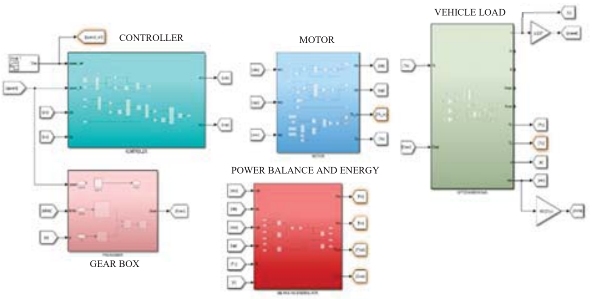

Fig. 3. Integrated simulation model

A. Motor Simulation Model

A PMSM motor simulation model, developed in Simulink on the basis of the Eqs. (6), (7) and (8), is presented in Fig. 4. At this model, v_{ds} and i_{qs} are inputs, and i_{ds} is the output.

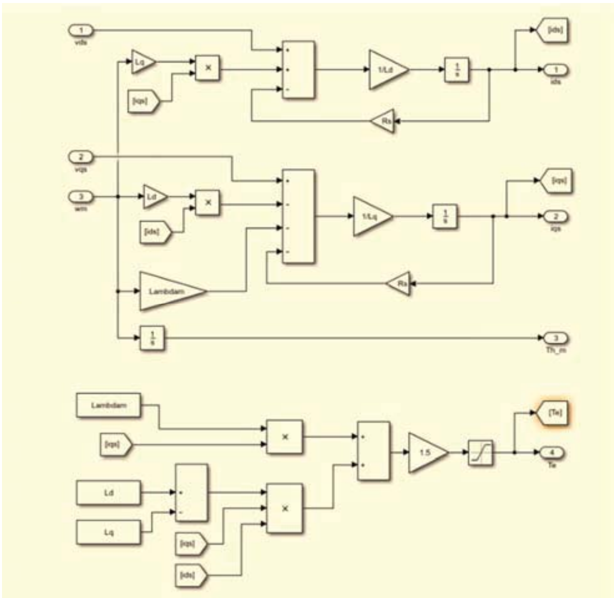

Fig. 4. Simulation model of PMSM motor

B. Model of the Controller

PI controller is used for vehicle speed control. The d-axis current is controlled to zero, so only the q-axis current produces torque. Kp and Ki gains are chosen so as to provide a critically damped step response, as well as a reasonable steady state error less than 2%. The simulation block diagram of the controller is shown in Fig. 5.

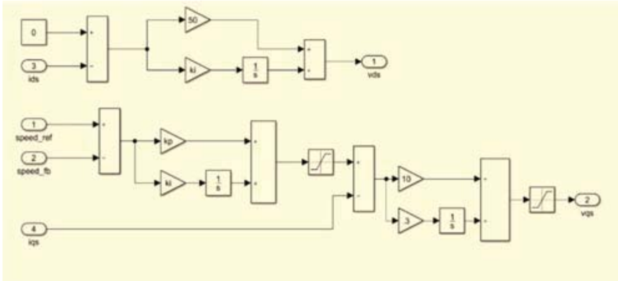

Fig. 5. Simulation model of PI Controller for speed and current

C. Load Modeling

The load model is developed based on the key parameters of the vehicle and the environmental conditions. The simulation model of vehicle load developed in Simulink (Fig. 6) is based on the mathematical model given in Section. II *B*.

Vehicle weight of 1500 kg is used on the basis of the passenger fleet averages. Although the viscous drag coefficient is a function of the vehicle aerodynamics, an average value of 0.5 is used. The tires and rotating parts weight is taken to be 50 kg, the tire loss coefficient is 0.01, A_{Front} = 3.7 m^2 and r_{tire} = 0.3 m.

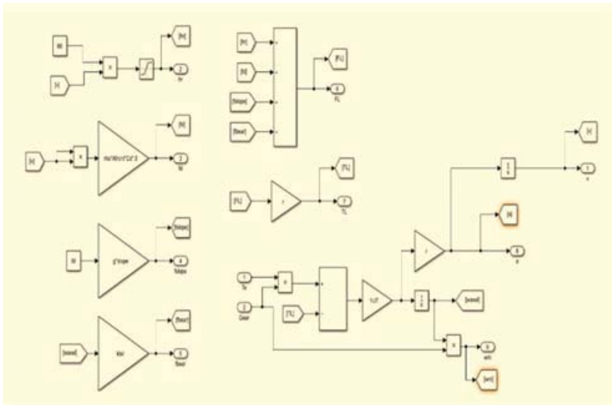

Fig. 6. Simulation model of vehicle load

D. Gear Modeling

The main objective of the simulation is to find a combination of gear ratios that will produce the most energy efficient drive cycle, and provide the highest possible mileage. For the basic model, nominal gear ratios are selected so that sufficient power can be generated and delivered to the wheels. In order to find the optimal gear ratio for the lowest energy consumption, a range for each gear is used. The range is selected so that the vehicle can provide reasonable torque and power to move under the loads. The simulation is made using random integers within the range of each gear and determining the energy used during the drive cycle.

A state model for this gear transition used in the simulation is shown in Fig. 7. The used model ensures that the engine speed does not exceed the set value of 5000 RPM. The model

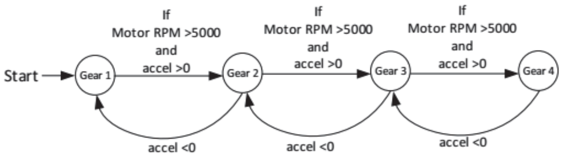

Fig. 7. Gear transition

also allows the both, up-shifting during acceleration, and down-shifting during deceleration.

IV. Simulation Results

One of the best ways to test a control system is to input a step function. Understanding the system's step response is important to determine how well the system is designed and tuned.

In this paper the simulation is performed using step function for vehicle speed as an input, not only to determine the parameters of the system in the setting of PID loop, but also to optimize the gear ratios.

The system step response for two distinctive cases is considered, i.e. in a case of acceleration of the vehicle, as well as in a case of vehicle decelerates. For the both cases, the diagrams of speed and acceleration of vehicle, the motor current, the torque, as well as the input and output power are determined (Figs. 8 and 9, respectively).

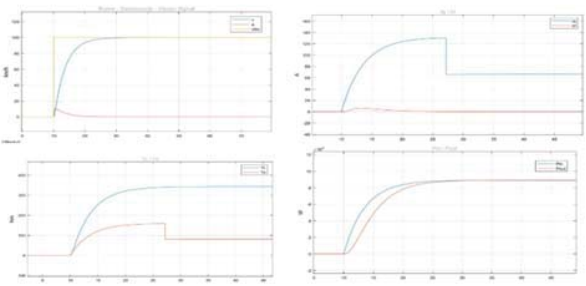

Fig. 8. System step response due to acceleration

Step response shows the settling time i.e. how much time it takes for the vehicle to achieve desired speed. Step response also shows the steady state error of the system control loop. According to the simulation results, the system settling time is 20 seconds, achieving steady state commanded input in this duration. As it was already noted, this is important to confirm that the PI gains have been correctly selected.

Initially, the motor torque is high due to low speed, and the input power drawn from the battery is high due to initial high load requirement, mainly due to the required acceleration. In the beginning the acceleration increases, since the vehicle starts from zero speed. As the speed reaches steady state, the acceleration decreases to zero, and input and output power become theoretically equal.

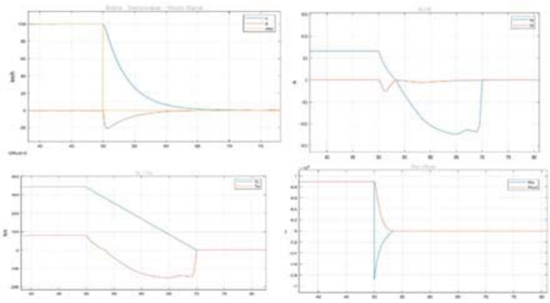

Fig. 9. System step response due to deceleration

When the set input speed is zero, the system goes into regenerative braking mode. During this time as the vehicle decelerates, and the speed goes down to zero, the mechanical system is putting power back into the battery. This can be clearly seen in the plot as the negative input power.

On the basis of the simulation results, it can be concluded that the system is performing well in a case of step input.

V. Conclusion

In this paper, the series-parallel HEV modeling based on multi-physics approach is presented. The system integrated simulation model, including the electric drive system, PI controller, vehicle load model, and gear box, is developed using Matlab/Simulink. Through a performed simulation the system step response is determined. The simulation results show that the system is performing well when step input is given, which confirms the model validity.

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