Plume Boundaries Extraction by Multiresolution and Least Squares Approximation

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Abstract – In this paper the problem of extracting plume boundaries is discussed. The plume rise of exhausted gases, containing a gas pollutant is investigated. The main goal is by using multiresolution and approximation to estimate the effective plume height from the plume central line in order to predict the plume behaviour.

I. Introduction

Many authors tried to define a precise algorithm for calculation of plume rise above the chimney at different conditions in the atmosphere. Accurate estimates of plume rise are required to predict the dispersion of continuous gaseous emissions [1]. The effective stack height is a very important characteristic, since it defines to a great extent the air pollution at the ground level [2]. The importance of the effective stack height makes its calculation one of the most important points in all mathematical models or software products intended for the air pollution assessment caused by existing or future sources of pollutants. The effective height is defined as a sum of the geometric height of the stack and the plume rise caused by the initial gas velocity and the gravitational force. The general situation is illustrated in Fig. 1.

This paper proposes the plume central line to be approximated through the approximated plume boundaries in the wavelet domain. There are many different techniques for edges detection in images published in the literature [3-5]. Edges in images can be mathematically defined as local singularities. Wavelet transform has received great attention in the last years, because it is especially suitable for time-frequency analysis [6], which is essential for singularity detection. With the growth of wavelet theory, the wavelet transforms have been found to be remarkable mathematical tools to analyze the singularities including the edges, and further, to detect them effectively.

In the paper, plume boundaries and plume central line are approximated by applying best fitting least squares approximation over the most important wavelet coefficients of the plume.

The paper is structured as follows. The wavelet theory is summarized in Section II. Section III presents the algorithm for plume boundaries extraction. The experimental results are presented in Section IV. Section V concludes the paper.

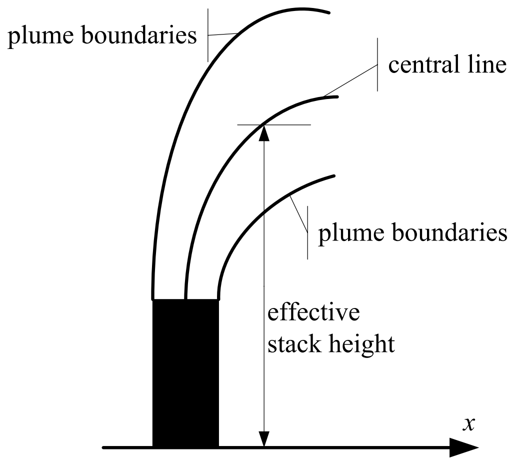

Figure 1. Plume rise in the atmosphere.

II. Wavelet Theory

The Discrete Wavelet Transform (DWT) decomposes a signal into a set of orthogonal components describing the signal variation across the scale [6]. The orthogonal components are generated by dilations and translations of a prototype function ψ, called mother wavelet.

In analogy with other function expansions, a function f is presented for each discrete coordinate t as a sum of a wavelet expansion up to certain scale J plus a residual term, that is:

$$f(t) = \sum_{j=1}^{J} \sum_{k=1}^{2^{-j}M} d_{jk}\psi_{jk}(t) + \sum_{k=1}^{2^{-J}M} a_{Jk}\phi_{Jk}(t) \qquad (1)$$

where ψ_{jk} and ϕ_{jk} denote wavelet and scaling function, respectively, the indexes j and k are for dilatation and translation, and a_{Jk} and d_{jk} are approximation and detail coefficients. The approximation coefficients a_{Jk} contain the signal identity while the detail coefficients d_{jk} can be processed for the purposes of denoising, compression, edge detection, etc.

Wavelet decompositions and multiresolution concepts are closely related to filter bank theory. For this reason, it is helpful to view the scaling and wavelet function as a low pass and high pass filters, $\mathbf{H_0}$ and $\mathbf{H_1}$, respectively. The wavelet

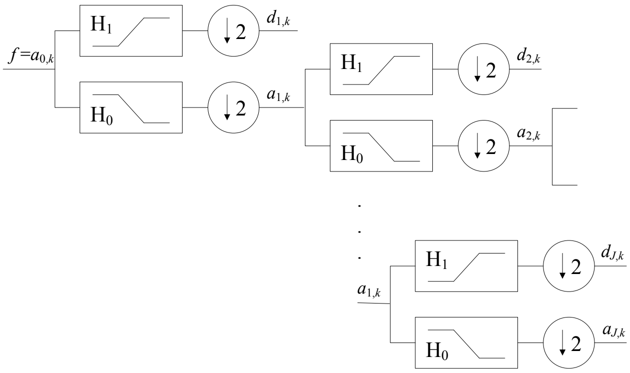

Figure 2. Discrete wavelet transform tree.

transform is applied to low pass results (approximations) as it is illustrated in Fig. 2.

The most popular form of wavelet-based filtering, wavelet shrinkage [7], is performed by weighting the corresponding detail wavelet coefficient by h_{ik} $(0 \le h_{jk} \le 1)$ and calculating the inverse wavelet transformation. Conventionally, the filtration is performed either by using "hard threshold" nonlinearity

$$h_{jk}^{(\text{hard})} = \begin{cases} 1, & \text{if } |d_{jk}| \ge \tau_j \\ 0, & \text{if } |d_{jk}| < \tau_j \end{cases} \qquad (2)$$

or by using "soft threshold" nonlinearity

$$h_{jk}^{(\text{soft})} = \begin{cases} 1 - \dfrac{\tau_j \operatorname{sgn}(d_{jk})}{d_{jk}}, & \text{if } |d_{jk}| \ge \tau_j \\ 0, & \text{if } |d_{jk}| < \tau_j \end{cases} \qquad (3)$$

where $\tau^{(k)}$ is user specified threshold for the k-th level details.

III. Boundaries Approximation

The main idea is to analyse and process the object of interest – chimney plume by using multiresolution in order to obtain its median line, important for calculating/predicting the plume direction. This is carried out through processing the plume's wavelet coefficients and afterwards dividing the coefficients into two segments by applying best fitting approximation in the least squares sense. The obtained two segments correspond to the plume boundaries, which afterwards can be approximated by applying again the best fitting operation individually over the two segments.

The procedure of extracting the plume boundaries from an RGB image, starts with converting the RGB image to YCbCr colour space, where Y is the luminance (intensity) component and Cb (blue chrominance) and Cr (red chrominance) are the blue-difference and red-difference chroma components, respectively. Then, the wavelet transform is applied over the Y component and the wavelet detail coefficients are filtered with (4) in order to keep only the most important coefficients. Afterwards, only the coefficients positions (x, y) are used in a polynomial, logarithmic, power or other approximation in the least squares sense, while the coefficients values are not in the focus. With the approximation, a temporary curve S_t that best fits coefficients positions is obtained and it is used in the process of plume boundaries segmentation to obtain two segments S_1 and S_2. Segments S_1 and S_2 could be obtained by comparing y coordinates of the filtrated wavelet coefficients and the curve S_t for each coordinate x. Once segments S_1 and S_2 are obtained, the best fitting operation is repeated separately for segments coefficients positions and as a result approximation curves S_{t1} and S_{t2} are obtained. They correspond to the plume boundaries.

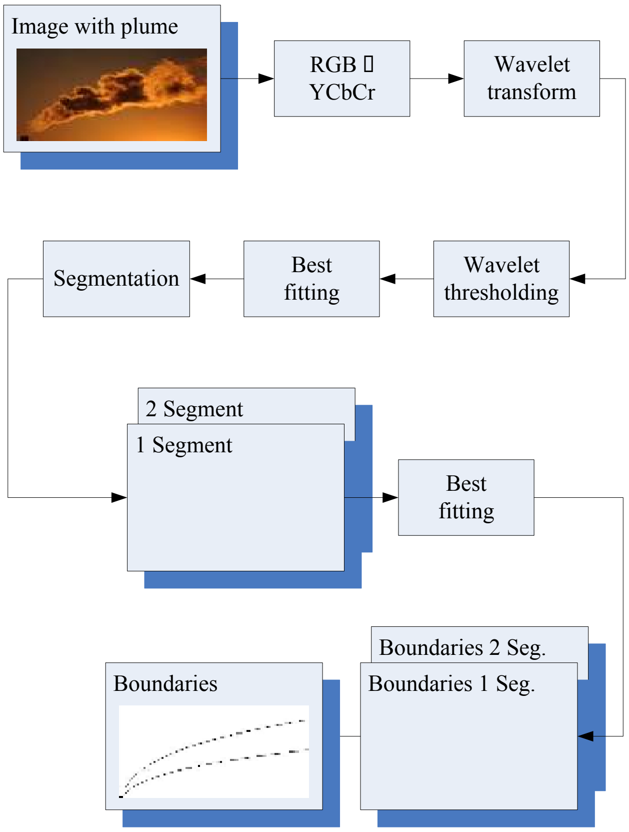

Figure 3. Block diagram of segmentation with the used algorithm.

The process of approximating plume boundaries can be summarized with the block-diagram shown in Fig. 3.

IV. Experimental Results

A number of experiments for approximating the plume boundaries are made with a dozen of images with different resolutions. Some of these images are shown in Fig. 4. The first part of the presented algorithm (approximation of plume centerline) was applied on all images and results are shown in Fig. 5. Approximation of the centerline is quite correct, but it is reasonable to calculate the plume boundaries only for well defined skewed cone shape of the plume visible shown on first three images.

The phases of the described process are illustrated in Fig. 6. The RGB image (Fig. 6a) is converted in YCbCr colour space. The haar wavelet transform is applied over the Y components and the most important pixels from the wavelet detail coefficients are kept (4% of pixels for the image in Fig. 6b). The obtained (x, y) positions take part in the best fit approximation in the least square sense. Experiments with

Figure 4. Experimental images.

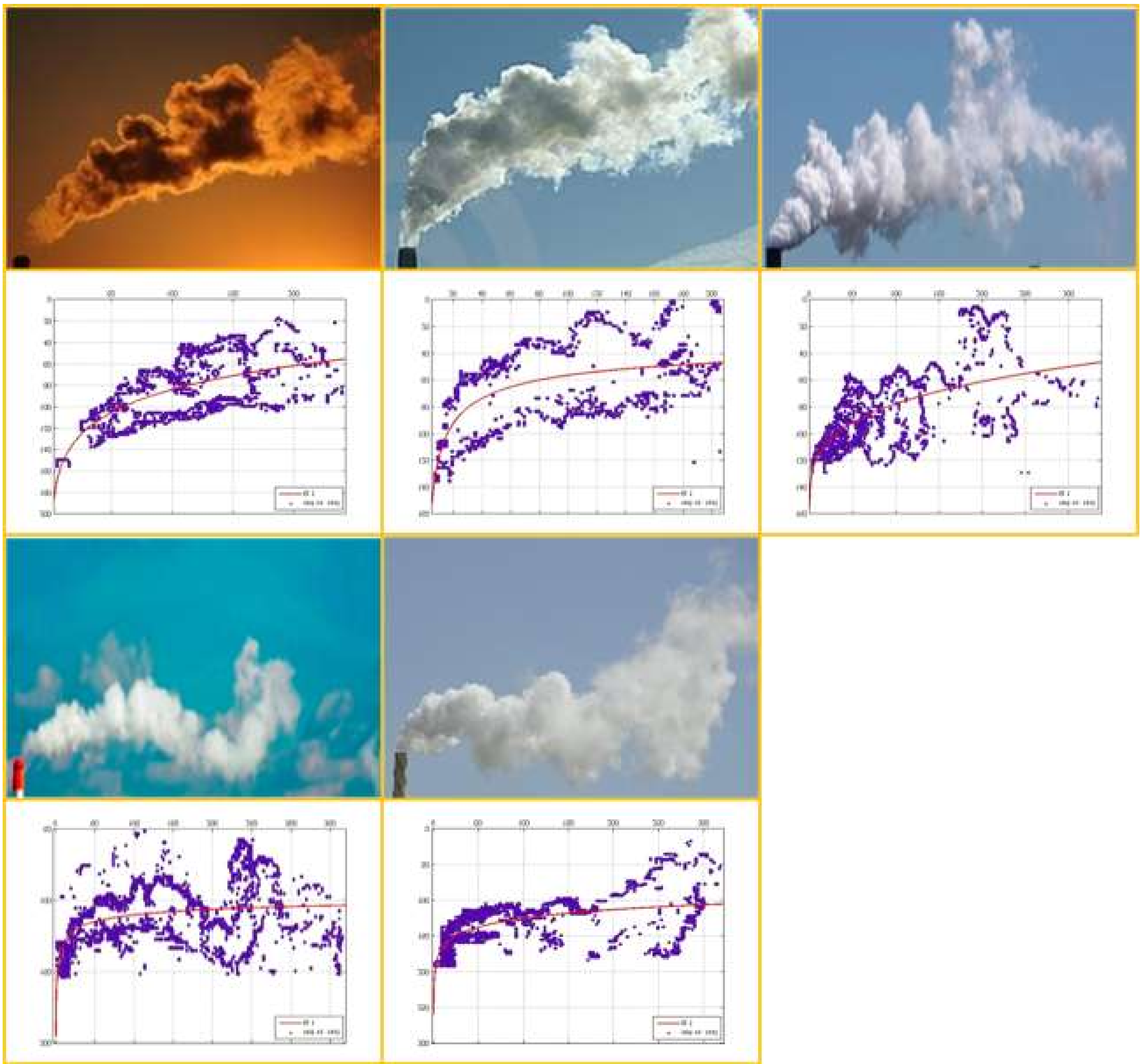

Figure 5. Results when the algorithm is applied over the experimental images from Fig. 4.

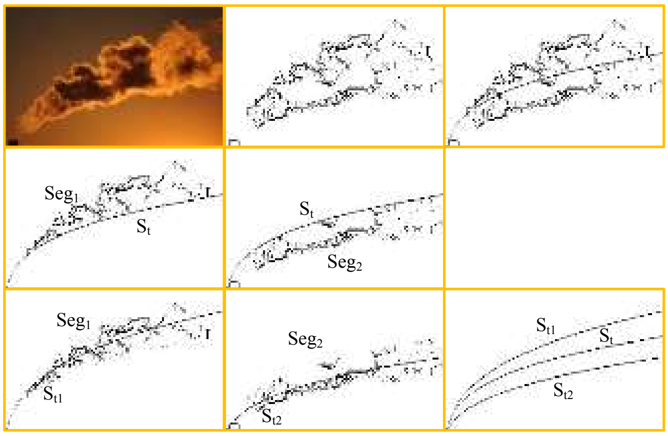

Figure 6. The process of segmentation through wavelet decomposition and approximation: (a) original image, (b) the most important wavelet coefficients, (c) curve that approximates the coefficients coordinates, (d-e) two segments, (f-h) approximated boundaries.

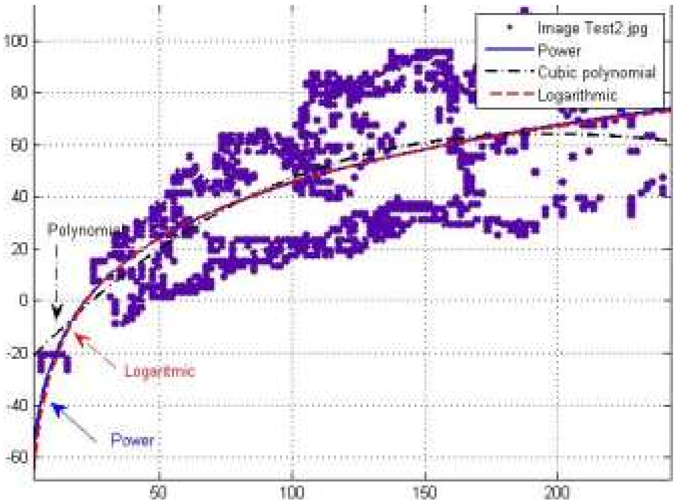

Figure 7. Best fitting curves in the least squares sense by using cubic polynomial, logarithmic and power approximations.

i. cubic polynomial ($f(x) = a_3x^3+a_2x^2+a_1x+a_0$) approximation,

ii. logarithm ($f(x) = a\ln b+c$) approximation and

iii. power ($f(x) = ax^b+c$) approximation

are carried out. Results of these three approximations over the image in Fig. 6a are shown in Fig. 7. Fig. 6c shows the temporary curve S_t that fits the coordinates of the wavelet coefficients by using the power approximation. The curve S_t is used to obtain two segments S_1 and S_2 (Fig. 6d-e) by comparing its *y* coordinates with the *y* coordinates of the filtrated wavelet coefficients (Fig. 6b) for each *x* coordinate. Next, the best fitting operation is applied again, now over the coefficients that belong to the segments S_1 and S_2 and as a result two curves, S_{t1} and S_{t2}, are obtained, that represents approximations of the plume boundaries (Fig. 6g-f).

V. CONCLUSION

The paper considers estimating plume central line by using wavelet transform and least squares approximation. The central line is important in order to calculate the effective stack height and to be able to predict the dispersion of continuous gaseous emissions.

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