Modelling of Stress–Strain Behaviour During Turning Operations

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***Abstract*—This paper presents a modelling of the stress–strain behaviour during the turning process in MATLAB. The material response is simulated under real machining conditions, including the transition from elastic to plastic deformation as governed by classical elasticity and yield-criterion theory. The resulting stress-strain curve is visualised as a dynamic diagram that clearly marks the yield point and delineates elastic and plastic regimes. The model demonstrates both theoretical relevance and practical applicability, serving as a foundation for parametric studies, process optimisation, and the development of advanced simulation tools in metal cutting and material forming.**

***Keywords*—Elastic deformation, Material flow, Hooke's law, Young's modulus of elasticity, Yield strength**

I. Introduction

In recent years, the rapid advancement of digital technologies has fundamentally transformed analytical approaches within mechanical and manufacturing engineering [1]. This transformation facilitates the application of sophisticated computational methods to better understand and optimise machining operations. Simulation-based methods now play a significant role in the analysis and optimisation of machining processes, enabling engineers to study complex physical phenomena under controlled conditions without the cost and constraints of physical experimentation [2].

Among the most widely used material removal processes, turning remains essential for producing precision components across diverse sectors, including automotive engineering, aerospace manufacturing, and microelectronics. Its ability to achieve tight dimensional tolerances and complex surface geometries renders it indispensable in both mass production and high-precision manufacturing contexts. Previous work, [3], [4], have addressed simulation-based approaches to related CNC machining processes, including linear interpolation motion and toolpath validation in milling.

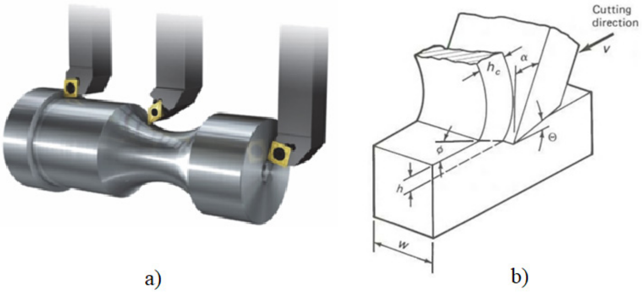
To further improve process performance, a deeper understanding of the material response under the mechanical forces induced during cutting is essential, especially when these responses involve irreversible structural changes within the workpiece material [5], [6]. Plastic deformation constitutes one of the principal physical phenomena occurring during turning. It refers to the permanent modification of the internal structure of a material subjected to external forces that exceed its elastic limit. This behaviour alters the geometry of the machined component and affects key mechanical properties such as residual stress distribution, hardness, and surface integrity. These modifications critically influence the functional performance and operational lifespan of the final product. Plastic deformation initiates when the internal stresses generated through mechanical interaction with the cutting tool surpass the material's yield strength. Beyond this limit, the material undergoes irreversible strain, resulting in permanent microstructural and energetic changes [7].

Despite the availability of general-purpose finite element tools, lightweight analytical models that directly couple the elastic-to-plastic transition with machining parameters in a transparent, educationally accessible framework remain uncommon. Existing MATLAB-based approaches often treat the mathematical formulation and the numerical implementation as separate concerns, limiting their utility for parametric studies and process planning. This paper addresses that gap by presenting a unified MATLAB model in which every computational step is directly derived from the governing analytical equations, making the connection between theory and simulation explicit and traceable.

This paper is organised into five sections. The introduction sets out the research background and motivation, drawing on relevant literature in mechanical engineering, CNC machining, turning operations, and mathematical modelling. The second section describes the physical and mathematical model used to capture stress–strain behaviour during turning. The third section covers the MATLAB implementation and presents the simulation results, while the fourth discusses what the results mean and where the model has its limitations. The paper concludes by summarising the key outcomes and outlining directions for future work.

II. Physical and Mathematical Model

The development of the MATLAB model is systematically structured to establish a direct correspondence between the theoretical framework introduced in this section and its numerical implementation. The mathematical formulation presented below establishes the complete set of equations for both the mechanical and thermal behaviour of the workpiece material during turning. The physical setup of the turning process is illustrated in Fig. 1.



a) b)

Fig. 1. a) Principle of turning, b) Chip formation.

As shown in Fig. 1, the turning process naturally involves complex interactions, including the fundamental mechanisms of turning [8] and chip formation [9]. As the cutting tool advances against a rotating cylindrical workpiece, it creates localised zones of elevated stress and temperature at the cutting tool–workpiece interface. These zones are characterised by significant plastic flow, thermal softening, and material shearing. Such conditions not only determine chip formation and tool wear but also impact surface finish, dimensional accuracy, and overall process efficiency [10].

The stress state within the cutting zone is multi-dimensional, comprising tangential (cutting), axial (feed), and radial (thrust) force components, each contributing to the intricate deformation patterns observed. Fig. 2 illustrates these force components [11]. When the combined stress exceeds the material's yield strength, it transitions from elastic to plastic behaviour, fundamentally altering its response to further external force [12].

The fundamental relationships governing the stress-strain behaviour during turning are introduced through a unified mathematical framework based on classical machining mechanics and elastic-plastic material modelling [13]–[16]. Within this framework, the normal stress acting on the material due to the cutting force can be expressed as:

$$\sigma_n = \frac{F_c}{f \cdot d} \tag{1}$$

where F_c is the cutting force (N), f is the feed rate (mm/rev), and d is the depth of cut (mm).

The shear stress resulting from friction at the tool–workpiece interface is given by:

$$\tau = \mu \cdot \sigma_n \tag{2}$$

where μ represents the friction coefficient.

The equivalent stress, representing the combined effect of normal and shear stresses, is expressed as:

$$\sigma_{eq} = \sqrt{{\sigma_n}^2 + \tau^2} \tag{3}$$

The transition from elastic to plastic deformation is governed by the yield criterion:

$$\sigma_{eq} \le \sigma_y \Rightarrow \text{Elastic behaviour} \tag{4}$$

$$\sigma_{eq} > \sigma_y \Rightarrow \text{Plastic behaviour} \tag{5}$$

where σ_y is the yield strength.

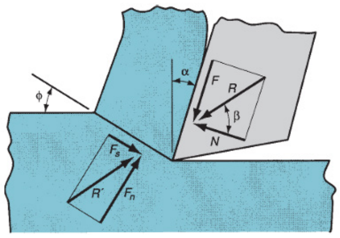


Fig. 2. Cutting forces in the cutting zone.

Once the yield strength is exceeded, the plastic strain is calculated using a linear elastic–plastic approximation:

$$\varepsilon_p = \frac{\sigma_{eq} - \sigma_y}{E} \tag{6}$$

where ε_p is the plastic strain, E is Young's modulus.

These analytical expressions form the mathematical foundation of the implemented MATLAB simulation model, ensuring a direct correspondence between theoretical mechanics and numerical simulation.

To understand and predict these phenomena, a comprehensive modelling framework is indispensable. Modelling provides a robust means to represent the interaction between material properties and machining parameters. In this regard, MATLAB serves as a powerful computational environment for simulating the cutting process and visualising the effects of various inputs. Through algorithmic development and numerical simulation, engineers can predict the stress-strain behaviour of materials under realistic cutting conditions. This approach facilitates the identification of critical points such as the yield point and enables detailed analysis of post-yield material behaviour, which is vital for effective process planning and control [17].

By utilising MATLAB's computational capabilities, simulation models can replicate the thermal and mechanical effects generated in the cutting zone. These models can produce stress-strain curves that illustrate both elastic and plastic deformation regimes. Such visualisations not only clarify the material's transition behaviour but also provide practical insight into how machining parameters influence the deformation process. The elastic region of the stress-strain curve is characterised by a linear relationship governed by Hooke's Law, where stress is directly proportional to strain through the modulus of elasticity. Accordingly, Hooke's Law is mathematically expressed as:

$$\sigma = E \cdot \varepsilon \tag{7}$$

where σ is the applied stress (Pa or MPa), E is Young's modulus representing material stiffness in (Pa) or (MPa), and ε is the strain, a dimensionless measure of deformation.

When the applied stress exceeds the yield strength, this linearity ceases, and plastic deformation occurs, manifesting as the nonlinear extension of the curve. Accurate interpretation of these diagrams is crucial for diagnosing machining outcomes. For example, a steep initial slope signifies high material stiffness, while an extended plastic region indicates ductility, an important characteristic for materials subjected to dynamic forces.

Furthermore, the model's results can reveal inefficiencies such as excessive cutting temperatures or suboptimal tool geometries that might be difficult to detect experimentally. The simulation also supports prediction of key performance indicators such as material removal rate, surface roughness, and tool lifespan, parameters central to contemporary manufacturing optimisation [18].

In addition to the mechanical stress-strain relationships, the model incorporates a simplified thermal analysis to estimate the temperature rise in the workpiece material during cutting. The power input P generated during machining is expressed as:

$$P = \frac{F_{c0} \cdot v}{60} \tag{8}$$

where F_{c0} is the initial cutting force, and v is the cutting speed.

The total heat generation Q is obtained over the machining time t:

$$Q = P \cdot t \tag{9}$$

The temperature rise T is then estimated using an energy balance approach:

$$T = \frac{Q}{mc} \tag{10}$$

where m is the heated mass. Although this formulation assumes uniform heat distribution and neglects heat losses, it provides a first-order approximation of thermal effects during cutting.

Together, equations (1)–(10) form the complete analytical foundation of the MATLAB simulation model. Each computational step in the implementation corresponds directly to one of these expressions, ensuring methodological consistency and transparency between the theoretical formulation and its numerical realisation.

III. COMPUTATIONAL MODELLING FRAMEWORK

The development of the MATLAB model is systematically structured to establish a direct correspondence between the theoretical framework introduced in Section II and its numerical implementation. The modelling procedure follows a sequential approach, where each computational step is derived from the analytical expressions (1)–(10), ensuring methodological consistency and transparency.

The modelling process begins with the definition of the fundamental material properties, namely the yield strength σ_y and Young's modulus E, which govern the transition between elastic and plastic behaviour. These parameters are essential for evaluating the conditions defined by (4) and (5), which distinguish the elastic and plastic regimes.

In parallel, the machining parameters are introduced, including the feed rate f, depth of cut d, cutting speed v, and friction coefficient μ. These parameters serve as inputs for the computation of stresses generated during the turning process.

The first step in the computational procedure is the determination of the normal stress acting on the material. This is calculated using (1), where the cutting force F_c is distributed over the effective cutting area defined by the product of feed rate and depth of cut. This formulation provides a simplified yet physically meaningful representation of the stress state within the cutting zone.

Subsequently, the shear stress component is evaluated according to (2), incorporating the influence of friction at the tool–workpiece interface. This step reflects the mechanical interaction between the cutting tool and the material, which significantly contributes to the overall deformation process.

The equivalent stress is then determined using (3), combining the normal and shear stress components into a single scalar quantity. This equivalent stress represents the effective driving force for material deformation and serves as the key parameter for assessing the onset of plasticity.

The transition from elastic to plastic behaviour is evaluated through the conditional criteria defined in (4) and (5). For each computational step, the model verifies whether the equivalent stress remains below or exceeds the yield strength. If the condition in (4) is satisfied, the material response is considered elastic, and no permanent deformation is accumulated. Conversely, when the condition in (5) is met, plastic deformation is initiated.

In the plastic regime, the corresponding plastic strain is calculated using (6), which defines a linear elastic–plastic relationship. This formulation assumes ideal plastic behaviour, where strain increases proportionally with the excess of stress beyond the yield limit. This simplification enables efficient computation while preserving the essential characteristics of material deformation.

The numerical implementation of this framework is realised through an iterative procedure in MATLAB. A loop structure is employed to simulate the progressive increase in cutting force, allowing the model to evaluate stress and strain evolution over a range of operating conditions. During each iteration, the stresses computed using (1)–(3) are updated, and the deformation state is determined using (4)–(6).

The computed values of equivalent stress and plastic strain are stored in arrays, forming a dataset that describes the material response throughout the simulation. This dataset is subsequently used to generate the stress–strain curve, where the elastic region follows the linear relationship defined by (7), while the plastic region reflects the deviation from linearity due to permanent deformation.

In addition to mechanical modelling, the simulation incorporates a simplified thermal analysis. The power input P during cutting is calculated based on the cutting force and cutting speed using (8), and the resulting heat generation Q is used to estimate the temperature rise in the material. Although this approach assumes uniform heat distribution, it provides a first-order approximation of thermomechanical coupling during the turning process.

The overall structure of the MATLAB model follows a modular design, where each stage of the computation corresponds to a specific physical phenomenon: stress evaluation, deformation analysis, and thermal estimation. This modularity enhances the clarity of the implementation and facilitates future extensions of the model, such as incorporating strain hardening, temperature-dependent material properties, or dynamic cutting effects.

To ensure interpretability and reproducibility, all computational steps are explicitly linked to the analytical expressions (1)–(10). This direct mapping between theory and implementation represents a key strength of the proposed approach, providing both conceptual clarity and practical applicability in the analysis of stress–strain behaviour during turning operations.

A. Measurement Considerations

In machining processes, quantities such as stress, strain and temperature are usually obtained through measurements, using different types of sensors and instruments. For example, cutting forces can be measured using dynamometers, strain can be estimated with strain gauges, while temperature in the cutting zone is often monitored using thermocouples or infrared sensors.

However, obtaining accurate measurements in the cutting zone is not straightforward. The conditions are complex, with high temperatures, large gradients and limited accessibility, which can affect both the accuracy and reliability of the measured data. Because of this, experimental measurements are often complemented with computational models.

The MATLAB model presented in this paper follows this approach by providing a simplified way to estimate the main physical quantities involved in the turning process. Although the model does not include all real effects, it allows the user to observe how changes in input parameters influence stress, strain and temperature. In this way, it helps in understanding the general behaviour of the system and the relationships between key variables.

From a practical measurement perspective, the results obtained from the model should be seen as approximate. In real conditions, measurements are affected by factors such as sensor accuracy, calibration, noise and local variations in the material. Despite these limitations, the model provides useful insight and can serve as a practical tool for interpreting trends and supporting further experimental analysis.

B. Modelling Assumptions

In order to ensure computational simplicity and clarity of interpretation, the model is based on a set of simplifying assumptions that allow the fundamental mechanisms of plastic deformation during turning to be captured in a controlled and transparent manner.

The material is assumed to exhibit ideal elastic–plastic behaviour, meaning that no strain hardening is considered once the yield strength is exceeded. In addition, the stress distribution is taken to be uniform across the cutting cross-section, which simplifies the representation of the mechanical state within the cutting zone.

From a thermal perspective, the model assumes uniform heat generation and distribution within the material, while heat losses due to conduction and convection are neglected. Similarly, the cutting forces are treated as quasi-static, thereby excluding dynamic effects such as vibrations or transient force fluctuations that may occur in real machining conditions.

Furthermore, the influence of tool geometry and tool wear is not explicitly incorporated into the model, and the material behaviour is considered independent of strain rate and temperature variations. These simplifications reduce the complexity of the simulation while maintaining its conceptual consistency.

These assumptions simplify the modelling approach and make the simulation easier to implement and analyse. However, they also affect the accuracy of the obtained results. As a result, the values predicted by the model may differ from those observed under real machining conditions. Therefore, the results should be interpreted as approximate and mainly useful for understanding the general behaviour of the system, rather than as exact values.

IV. Results and Discussion

The MATLAB script operates through a systematic sequence of computations designed to model the mechanical and thermal behaviour of the workpiece material subjected to machining forces. Initially, the script defines a set of fundamental parameters representing both material properties and machining conditions. These include the yield strength (y_s), elastic modulus (E), feed rate (f), depth of cut (d), cutting speed (v), and friction coefficient (μ), as well as physical constants such as material density (ρ), specific heat capacity (c), and machining time (t). These parameters correspond directly to the variables used in the analytical expressions (1)–(7) and establish the baseline conditions for the subsequent iterative analysis.

The simulation uses the initial values given in Table I. The initial cutting forces are set to F_{c0} = 500 N, F_{t0} = 100 N, and F_{r0} = 50 N, as summarised in Table I. During each iteration, the script recalculates the primary cutting force F_c, thrust force F_t, and radial force F_r, thereby simulating progressively increasing loading conditions. Within each iteration, the normal stress σ_n is calculated according to (1), by distributing the cutting force over the effective cutting area. The shear stress component τ is then determined using (2), incorporating the influence of friction at the tool–workpiece interface. The resultant equivalent stress σ_{eq} is computed using (3), combining the normal and shear stresses into a single measure representing the overall deformation-driving mechanism.

TABLE I. Input Parameters for the MATLAB Model

Parameter	*Symbol*	*Value*	*Unit*
Yield strength	y_s	300	MPa
Elastic modulus	E	2×10^5	MPa
Feed rate	f	0.1	mm/rev
Depth of cut	d	2	mm
Cutting speed	v	150	m/min
Friction coefficient	μ	0.3	–
Material density	ρ	7800	kg/m³
Specific heat capacity	c	500	J/(kg·K)
Machining time	t	60	s
Initial cutting force	F_{c0}	500	N
Initial thrust force	F_{t0}	100	N
Initial radial force	F_{r0}	50	N

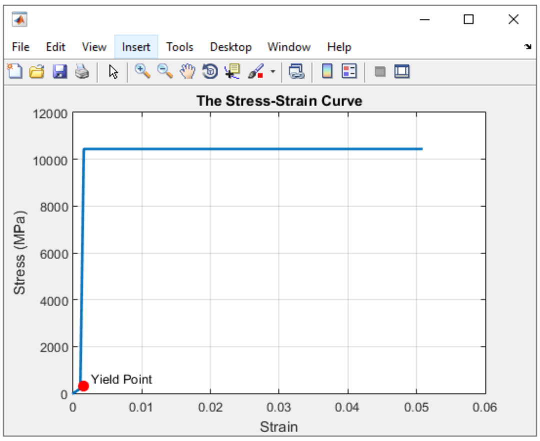


Fig. 3. The stress-strain curve.

To evaluate the onset of plastic deformation, the script employs a conditional expression consistent with the criteria defined in (4) and (5). Plastic strain ε_p is computed only when the equivalent stress exceeds the yield strength, following the formulation given in (6). This approach ensures that deformation remains purely elastic below the yield limit and transitions to plastic behaviour once the critical stress threshold is exceeded.

The stresses and strains computed in each iteration are stored in arrays (sr and ep), forming a dataset that describes the evolution of the material state throughout the simulation. This dataset is subsequently used for graphical and numerical analysis.

Following the mechanical computations, the script performs a simplified thermal analysis based on the power-heat-temperature formulation defined in (8)–(10). The temperature rise is estimated using an energy balance approach, assuming uniform heat distribution within the workpiece.

For graphical representation, the script generates the stress–strain curve (Fig. 3). The elastic region follows the linear relationship defined by (7), while the plastic region is represented by a deviation from linearity beyond the yield point. The yield point is explicitly marked, clearly indicating the transition from elastic to plastic behaviour.

Finally, the script outputs key quantitative results, including the final stress value, the accumulated plastic strain, and the estimated temperature rise. The final resultant stress reaches approximately 10440.31 MPa, while the plastic strain at maximum applied force is 0.0507. The estimated temperature increase during cutting is 10.68°C.

From a physical standpoint, the obtained stress values are higher than typical values observed in real machining processes. This discrepancy is primarily due to the simplified assumptions of the model, particularly the concentration of force over a limited area as defined in (1), and the absence of mechanisms such as stress redistribution and strain hardening. Similarly, the relatively low temperature rise reflects the simplified thermal model, which does not account for heat partitioning between the tool, chip, and workpiece or for localised heating effects.

A comparison with typical industrial turning conditions further illustrates these limitations. In real machining operations, stress levels in the primary shear zone generally range from several hundred MPa up to approximately 1–2 GPa, depending on the material properties and cutting parameters. In contrast, the significantly higher stress values predicted by the present model indicate that the simplified formulation in (1)–(3), particularly the assumption of uniform stress distribution over a reduced contact area, leads to an overestimation of the effective stress. Similarly, the estimated temperature rise of approximately 10.68°C is considerably lower than values observed in practice, where temperatures frequently reach several hundred degrees Celsius due to intense plastic deformation and frictional heating at the tool–chip interface [19], [20].

These discrepancies highlight the influence of the simplifying assumptions adopted in the model, including the neglect of strain hardening, strain-rate sensitivity, heat partitioning, and localised deformation effects. In real cutting conditions, deformation is highly non-uniform and concentrated within narrow shear zones, while thermal energy is unevenly distributed between the tool, chip, and workpiece. Despite these limitations, the model successfully captures the fundamental trends governing the turning process, particularly the transition from elastic to plastic behaviour and the qualitative dependence of stress on machining parameters.

From an engineering standpoint, the results can therefore be interpreted as indicative of relative behaviour rather than absolute values. This makes the model particularly useful for parametric studies, sensitivity analysis, and preliminary process design, where the goal is to understand how variations in cutting conditions influence material response. Furthermore, the ability to rapidly simulate different scenarios without experimental setup provides practical advantages in process optimisation, tool selection, and educational applications, where visualisation of stress–strain behaviour plays a significant role in understanding material mechanics during turning operations.

A key advantage of this approach is its flexibility. By modifying input parameters such as feed rate, cutting speed, and depth of cut, users can explore different machining scenarios and analyse their influence on deformation behaviour. This makes the model suitable for preliminary analysis, process optimisation, and educational purposes.

V. Conclusion

This paper presents a MATLAB-based model for analysing stress–strain behaviour during turning operations. The proposed framework enables the evaluation of both mechanical and thermal responses of the workpiece material under varying machining parameters, including cutting speed, feed rate, depth of cut, and friction conditions. The model allows identification of stress states, detection of the elastic–plastic transition, and estimation of plastic strain and temperature rise.

The generated stress–strain curves provide a clear interpretation of material behaviour, supporting the selection of cutting conditions that reduce tool wear and maintain dimensional accuracy. Due to its modular structure, the model can be easily adapted for parameter studies, process optimisation, and preliminary design analysis.

In addition to its engineering application, the model is suitable for educational use, offering a simple way to demonstrate fundamental concepts in machining and material behaviour through interactive simulation.

Future work will focus on improving model accuracy through the inclusion of strain-rate effects, temperature-dependent properties, and experimental validation, as well as extending the approach to other machining processes.

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