Toolpath Validation in CNC Milling: A Mathematical Model for Linear Interpolation

Violeta Krcheva[1], Stojance Nusev[1] and Gordana Janevska[1]

Abstract – **This paper demonstrates the integration of mathematical modelling with CNC milling by employing Newton's interpolation formula. A detailed model is developed and implemented in Matlab, facilitating a better understanding of CNC milling dynamics. This integration enhances precision and efficiency by validating cutting toolpath accuracy and reducing machining errors.**

Keywords – **CNC Interpolator, CNC Simulator, Data points, Matlab, Interpolating Polynomial.**

I. Introduction

Mathematical modelling is a fundamental approach for analysing and solving engineering problems in the field of mechanical engineering. Prior to conducting the analysis, it is imperative to transform the engineering problems into mathematical expressions comprising equations, functions, and relevant variables. The process of transforming the mathematical expressions into mathematical models is commonly referred to as mathematical modelling, or, briefly, modelling [1].

The purpose of modelling is to identify solutions in the form of functions that align with specified equations. This involves a precise analysis of their fundamental characteristics, leading to the analytical determination of values and graphical representation. By integrating theory with practical applications, the modelling process enhances understanding of engineering problems and drives efficiency and advancements in machining systems and processes [2].

Machining processes consist of a spectrum of machining operations commonly used in manufacturing and fabrication to shape, cut, or remove material from a workpiece with precision and repeatability. These processes are integral to various industries, including automotive, aerospace, electronics, and beyond, where precise dimensional tolerances and surface finish quality are paramount [3].

In industries reliant on precision engineering, CNC machine tools play a pivotal role. These sophisticated machine tools operate with exceptional accuracy and consistency under computerised control, offering micrometre-level precision. Enhanced by CNC simulators, operators gain insights into the complexity of cutting toolpath generation, thereby empowering them to optimise the toolpaths for improved efficiency and accuracy [4].

The CNC simulator and CNC interpolator are integral components of the CNC machine tool systems, interconnected through the exchange of G-code instructions. The CNC interpolator plays a crucial role in this connection, as it interprets the G-code instructions specifying the cutting tool movements. Subsequently, it generates the requisite control signals to drive the machine's axes, determining both the speed and direction of motion for each axis. This process ensures precise cutting tool positioning and motion during machining operations [5].

The CNC simulator functions distinctively from the CNC interpolator, as it computationally simulates the cutting tool movements and machining processes. This computational simulation affords operators the capacity to not only visualise but also analyse the complexities of the machining process. This capability extends beyond the operational phase of the machine tool, offering insights into cutting toolpaths, collision detection, and the overall efficacy of the processes [6].

CNC machine tools employ interpolation to control the cutting tools along predetermined toolpaths within a Cartesian coordinate system. Various interpolation forms, including linear (Fig. 1a), circular (Fig. 1b and Fig. 1c), and helical (Fig. 1d), are integral to CNC machining. Specifically, in linear interpolation, the CNC machine tool directs the cutting tool along a straight path between two defined points on the workpiece [7].

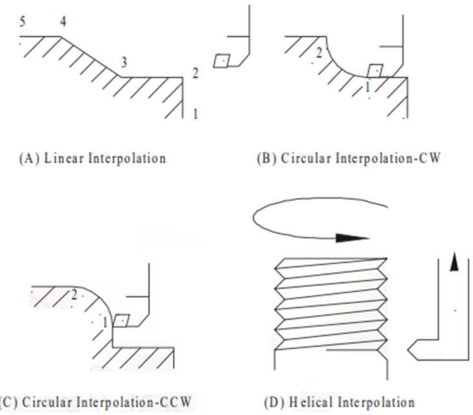

Fig. 1. Various types of interpolation [8]

One method to understand the function of the CNC interpolator and the movement of the cutting tool is by utilising Newton's linear interpolation within Matlab. Matlab is sophisticated computational software that enables engineers and machinists to visualise the results generated by the CNC interpolator and simulator in contemporary CNC machine tools [9].

In Matlab, Newton's linear interpolation formula is employed to construct degree-one (linear) interpolation polynomials that approximate the contours or shapes of workpieces. These polynomials facilitate the generation of cutting toolpaths that conform to specified requirements,

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ensuring that the resulting machined components achieve precise tolerances and quality standards [10].

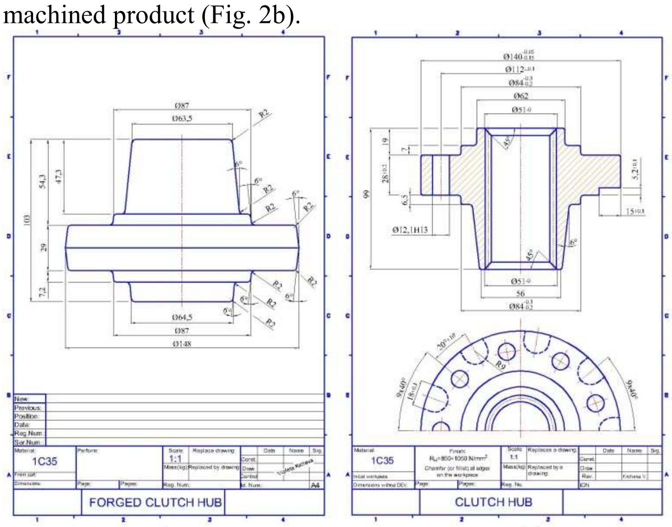
The integration of CNC milling with mathematical modelling using Newton's interpolation in Matlab marks a significant advancement in CNC machining technology. This approach enhances precision, efficiency, and the capability to create complex components, leading to innovative applications across various machining operations [11].

The development of a mathematical model significantly enhances the understanding of the operational complexities of a CNC simulator, which is a key component of modern CNC machine tools. This comprehension is vital for elucidating CNC machining processes and defining their characteristics, terms that are mentioned in relevant literature (e.g., [1]–[12]). The paper is structured into four sections: an introduction, a research explanation, a results discussion, and concluding remarks with potential applications.

II. Research

This research examines a segment of the clutch hub machining process specifically designed for a CNC milling operation. The clutch hub is a vital component of a vehicle's clutch system, serving as the central element of the clutch disc assembly in manual transmission vehicles.

To precisely analyse the milling operation, two-dimensional drawings have been created using CAD systems, particularly AutoCAD software. These detailed drawings accurately represent the machining process, illustrating the progression from the initial forged clutch hub (Fig. 2a) to the final machined product (Fig. 2b).



a) b)

Fig. 2. a) Initial forged clutch hub, b) Clutch hub [12]

CNC milling is an advanced machining operation that utilises a rotating cutting tool to precisely remove material from a workpiece. This technique is crucial in manufacturing due to its ability to produce intricate and accurate components. Specifically, groove milling, a subset of CNC milling, involves creating narrow, linear channels or slots on the material's surface. These grooves are essential in various applications, including the manufacturing of clutch hubs.

In the context of machining a clutch hub, CNC milling is employed to create precise grooves in the workpiece, as depicted in Figure 3. The detailed dimensions and proportions of the grooves to be milled into the workpiece are provided, offering a comprehensive view of its design specifications.

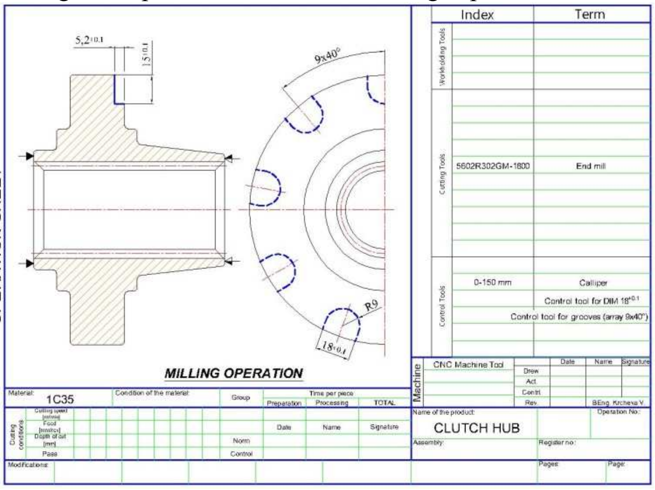


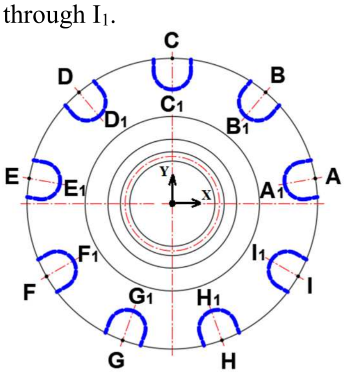
Fig. 3. Milling operation [12]

The milling operation begins with the careful alignment and secure clamping of the workpiece on the CNC milling machine. Driven by computer numerical control (CNC) technology, the rotating cutting tool systematically removes excess material to machine the grooves to exact specifications, which is crucial for the proper functioning of the clutch hub.

The operation begins with the secure clamping of the workpiece into the CNC machine tool. An appropriate milling cutter, such as the 5602R302GM-1800 end mill (Fig. 4) produced by ZCC-CT, is then selected for the operation. As the end mill rotates and traverses the designated path, it precisely cuts the groove into the workpiece. Achieving the required groove depth and width may necessitate multiple passes of the end mill.

Fig. 4. The end mill [12,13]

To specify the end mill's exact toolpath, Figure 5 depicts a 2D model along with a series of coordinates labelled A through I_1.



Coordinates (x;y)	
A (68.9;12.2)	A_1 (54.2;9.6)
B (45;53.6)	B_1 (35.4;42.1)
C (0;70)	C_1 (0;55)
D (-45;53.6)	D_1 (-35.4;42.1)
E (-68.9;12.2)	E_1 (-54.2;9.6)
F (-60.6;-35)	F_1 (-47.6;-27.5)
G (-23.9;-65.8)	G_1 (-18.8;-51.7)
H (23.9;-65.8)	H_1 (18.8;-51.7)
I (60.6;-35)	I_1 (47.6;-27.5)

a) b)

Fig. 5. a) Toolpath, b) Coordinates of the end mill toolpath

These coordinates define the exact toolpath through linear interpolation, where the end mill tool traverses straight lines between each point, ensuring smooth transitions from point A to point I_1.

The linear interpolated motion consists of a sequence of straight-line movements between specified points. At each designated point, the end mill alters its direction as it travels along the straight segment connecting these consecutive points. This approach ensures precision and control in defining the toolpath, which is essential for achieving the desired geometry of the workpiece.

The analysis of linear interpolated motion utilises Newton's linear interpolation formula to estimate function values at specific points within the range of a discrete set of known data points. It is a particular case of polynomial interpolation, where the polynomial used for interpolation is of the first degree, or linear.

In this approach, it is assumed that the values between two consecutive data points can be approximated by a straight line. Newton's linear interpolation formula is used to construct this line and estimate intermediate values. The formula is derived by constructing a linear polynomial that passes through two known points, ensuring that the interpolated values lie on this straight line.

To compute the interpolating polynomial within these linear segments, the linear interpolation formula is applied:

$$P_i(x) = y_i + (x - x_i) \cdot \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \quad (1)$$

In this equation, $P_i(x)$ represents a linear function derived from the interpolation formula, estimating the dependent variable *(y)* at the point *(x)* based on known values y_i and y_{i+1} at x_i and x_{i+1}, respectively. This formula constructs a straight line segment between points (x_i, y_i) and (x_{i+1}, y_{i+1}), estimating y for any x within this interval. If x_i and x_{i+1} are identical, no interpolation is needed due to the absence of an interval.

III. Results and Discussion

This section details the results of linear interpolation between specified points using Newton's linear interpolation formula. The process involved several key steps: identifying the intervals and the points, and applying the interpolation formula. The results are organised in Table 1, which includes columns for interval points, the starting and ending coordinates of each interval, and the interpolating polynomial within each interval.

Initially, the exact coordinates (x_i, y_i) and (x_{i+1}, y_{i+1}) for the interpolation points were determined. These points are the known values between which interpolation occurs. Next, the intervals between consecutive points were calculated. Newton's linear interpolation formula was then used to estimate values between the given points. Using equation (1), the interpolating polynomial P(x) for any x within the interval $[x_i;x_{i+1}]$ was computed, as shown in Table 1. If the x-values within the interval $[x_i;x_{i+1}]$ were the same, interpolation was not performed, and the y-value stayed constant, indicating direct movement along the axis without additional calculations.

Table I
Results from the Linear Interpolation

Intvl.	x_i	x_{i+1}	y_i	y_{i+1}	$P_i(x)$
[A;A₁]	68.9	54.2	12.2	9.6	0.2x
[A₁;A]	54.2	68.9	9.6	12.2	0.2x
[A;B]	68.9	45	12.2	53.6	132−2x
[B;B₁]	45	35.4	53.6	42.1	−0.3+1.2x
[B₁;B]	35.4	45	42.1	53.6	−0.3+1.2x
[B;C]	45	0	53.6	70	70−0.4x
[C;C₁]	0	0	70	55	/
[C₁;C]	0	0	55	70	/
[C;D]	0	−45	70	53.6	70+0.4x
[D;D₁]	−45	−35.4	53.6	42.1	−0.3−1.2x
[D₁;D]	−35.4	−45	42.1	53.6	−0.3−1.2x
[D;E]	−45	−68.9	53.6	12.2	132+2x
[E;E₁]	−68.9	−54.2	12.2	9.6	−0.2x
[E₁;E]	−54.2	−68.9	9.6	12.2	−0.2x
[E;F]	−68.9	−60.6	12.2	−35	−380−6x
[F;F₁]	−60.6	−47.6	−35	−27.5	0.6x
[F₁;F]	−47.6	−60.6	−27.5	−35	0.6x
[F;G]	−60.6	−23.9	−35	−65.8	−86−0.8x
[G;G₁]	−23.9	−18.8	−65.8	−51.7	0.3+2.8x
[G₁;G]	−18.8	−23.9	−51.7	−65.8	0.3+2.8x
[G;H]	−23.9	23.9	−65.8	−65.8	−65.8
[H;H₁]	23.9	18.8	−65.8	−51.7	0.3−2.8x
[H₁;H]	18.8	23.9	−51.7	−65.8	0.3−2.8x
[H;I]	23.9	60.6	−65.8	−35	−86+0.8x
[I;I₁]	60.6	47.6	−35	−27.5	−0.6x
[I₁;I]	47.6	60.6	−27.5	−35	−0.6x

Moreover, the coordinates of the designated points were entered into a Matlab script to formulate a mathematical model employing Newton's formula for linear interpolation. This procedure required the creation and execution of Matlab code to develop the model. The script takes as inputs the x and y coordinates of the points, uses Newton's linear interpolation formula to estimate intermediate values, and subsequently produces a graphical plot to visually depict the interpolation. To conserve space, the Matlab code is depicted in Figure 6 below.

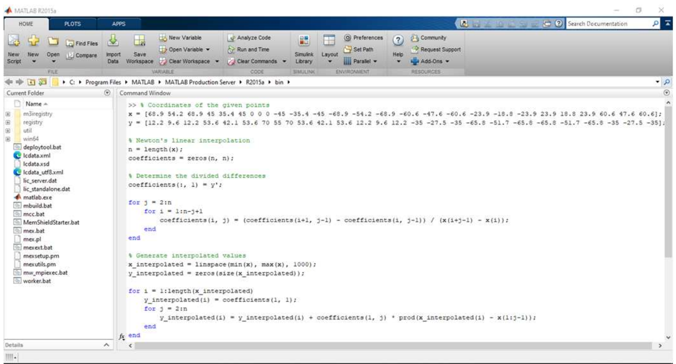


Fig. 6. The Matlab programming environment

Following the implementation of the interpolation model, Matlab code is employed to generate a graphical representation of the results. The output, illustrated in Figure

7, presents the linear interpolation applied to the milling operation, as indicated by the coordinates in Figure 5b.

Specifically, Figure 7 provides a detailed graphical representation of the linear interpolation utilised in CNC milling. This approach facilitates precise adjustments and enhancements throughout the operation, which are essential for achieving the required accuracy and quality in the final machined product. By utilising the coordinates from Figure 5b, the interpolation model converts these points into a smooth, continuous curve that delineates the milling toolpath throughout the CNC milling operation.

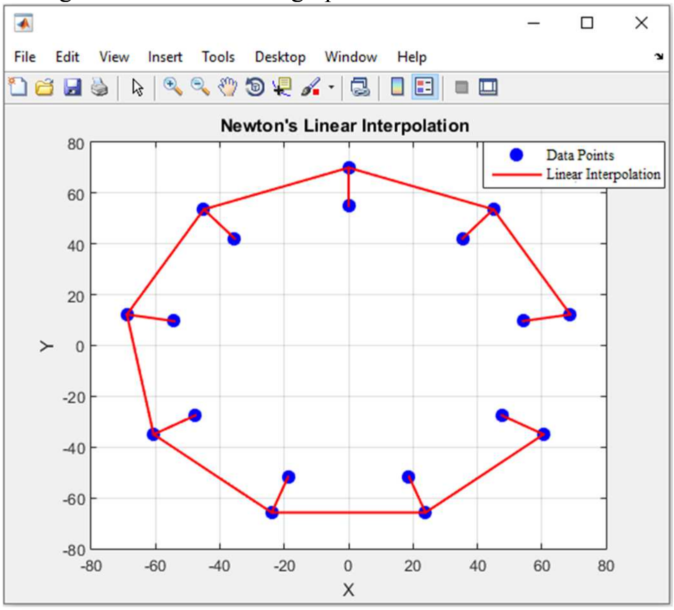


Fig. 7. The graphical solution

The graphical solution serves several key functions. Initially, it verifies the proper application of the interpolation model by showing a smooth transition between points that aligns with the intended milling toolpath, confirming the model's accuracy. It also provides engineers with insights into the model's performance, facilitating improvements within the CNC milling strategy. Additionally, it aids in optimising the CNC milling operation by illustrating the relationship between coordinates and the interpolation curve, helping to identify discrepancies and enhance machining accuracy, thereby reducing machining errors and improving efficiency.

IV. Conclusion

This paper explores a mathematical model of linear interpolation motion in CNC milling, employing Newton's linear interpolation formula. By devising and implementing this model in Matlab, the research offers critical insights into the dynamic behaviour of CNC milling machine tools during milling operations. The study highlights both theoretical and practical benefits for CNC machining.

Utilising Newton's linear interpolation formula to construct this model demonstrates its efficacy in depicting and predicting CNC milling machine tool motion. This approach effectively represents linear interpolation motion, which is vital for improving both the accuracy and efficiency of CNC milling operations. The Matlab implementation enhances its practical value by providing graphical solutions and visualisations, thereby deepening the understanding of the model's behaviour and its real-world implications for CNC milling.

The research underscores the significance of mathematical modelling in CNC machining, especially for comprehending and optimising the dynamic performance of CNC milling machine tools. The insights derived from this model can guide future developments in CNC technology, enhance milling precision, and support advancements in automated machining.

In conclusion, this linear interpolation model represents a substantial contribution to CNC milling, offering a solid foundation for both theoretical and practical applications. As CNC technology continues to advance, the insights from this model will be crucial for deepening the understanding of CNC milling and improving the performance of CNC milling machine tools. Future research may expand this model to explore more sophisticated interpolation approaches, incorporate additional factors affecting CNC machining, and further develop the mathematical framework to enhance CNC machining capabilities.

References

[1] E. Kreyszig, *Advanced Engineering Mathematics*, 10th ed., Hoboken, John Wiley & Sons, 2011.

[2] D. G. Zill, *Advanced Engineering Mathematics*, 6th ed., Burlington, Jones & Bartlett Learning, 2018.

[3] E. M. Trent, and P. K. Wright, *Metal Cutting*, 4th ed., Oxford, Butterworth-Heinemann, 2000.

[4] S. Kalpakjian and S. R. Schmid, *Manufacturing Engineering and Technology*, 6th ed., Singapore, Prentice Hall, 2009.

[5] M. P. Groover, *Fundamentals of Modern Manufacturing: Materials, Processes, and Systems*, 4th ed., Hoboken, John Wiley & Sons, 2010.

[6] G. Israeli, S. Mann, S. Bedi and A. S. Jawanda, "Numerical Verification of CNC Machine Simulations". Computer-Aided Design and Applications. vol. 8, no. 4, pp. 507-518, doi:10.3722/cadaps.2011.507-518, 2011.

[7] P. Petráček, P. Fojtu, T. Kozlok and M. Sulitka, "Effect of CNC Interpolator Parameter Settings on Toolpath Precision and Quality in Corner Neighborhoods". Applied Sciences. vol. 12, no. 19, doi:10.3390/app12199496, 2022.

[8] P. Radhakrishnan, S. Subramanyan and V. Raju, *CAD/CAM/CIM*, 3rd ed., New Delhi, New Age International Publishers, 2008.

[9] K. Evans, *Programming of CNC Machines*, 4th ed., South Norwalk, Industrial Press, 2016.

[10] S. Suh, S. Kang, D. Chung, I. Stroud, *Theory and Design of CNC Systems*, London, Springer-Verlag, 2008.

[11] D. E. Kandray, *Programmable Automation Technologies: An Introduction to CNC, Robotics ans PLCs*, New York, Industrial Press, 2010.

[12] V. Krcheva, S. Nusev and G. Janevska, "Simulation of Linear Interpolation Motion in CNC Machining". 59th International Scientific Conference on Information, Communication and Energy Systems and Technologies (ICEST). pp. 1-4. doi: 10.1109/ICEST62335.2024.10639790, 2024.

[13] (2023) ZCC Cutting Tools Europe GmbH, Main Catalogue. [Online]. Available: https://www.zccct-europe.com/en/tools/milling-cutters/