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MATHEMATICAL MODELLING OF FAR-INFRARED VACUUM DRYING OF APPLE SLICES

by

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In this study, a mathematical model of far-infrared vacuum drying of shrinkage body is presented. The system of two coupled PDE for heat and mass transfer with appropriate initial and boundary conditions are solved numerically with used of the finite difference method. On the basis of the numerical solutions a computer program for calculation of temperature profiles, transient moisture content, mid-plane temperature, and the volume averaged moisture content changes for different drying regime was developed. For verification of a mathematical model a series of numerical calculations were carried out with experimental conditions similar to those in the realized experiments of far-infrared vacuum drying of apple slices. Very good agreement between the experimental and numerical temperature and moisture content changes during the drying was obtained.

Key words: *mathematical modelling, far-infrared, vacuum, drying, apple*

Introduction

The mathematical modelling of the drying process of food materials is important for scientific and engineering calculations. The existing mathematical models which are used to model the drying process can be classified into several groups:
– first group consists of models that are used for modeling of drying kinetic curves using the phenomenological models *i. e.* using thin-layer drying equation [1-3]; these models can be categorized as theoretical, semi-theoretical, and empirical,
– in the second group are the so-called diffusion models that assume conductive heat transfer for energy and diffusive transport for moisture; in the case when there is available experimental data of drying kinetics, diffusion equations based on the second Fick's law is used for modeling of mass transfer; in this case, the diffusivity is the main mechanism for mass transport; then, the equation of second Fick's law can be solved analytically [4, 5] or numerically [6, 7] depending on whether the diffusivity is taken as a constant or variable value,

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−   in the third group are categorized models which are based on the numerical solution of the equation of second Fick's law with constant or variable value of diffusivity, whereby in the calculation are taken outside the mass transport and shrinkage of materials [7, 8],
−   the fourth group consists of models that are based on the simultaneous processes of heat and mass transfer; the modelling of drying process is defined by a system of two coupled PDE of second order with appropriate initial and boundary conditions, and the model take into account or not the shrinkage of the dry material [9-14],
−   in the fifth group are classified models that are based on the theory of porous media, and when it is applied equilibrium approach [15, 16], and
−   the sixth group consists of models based on the theory of non-equilibrium approach [17, 18].

The objectives of this study were: development of mathematical model which will used for modeling far-infrared vacuum drying of shrinkage body and verification of the mathematical model with comparison between experimental and predicted values of temperature and volume-averaged moisture content changes during far-infrared vacuum drying of apple slices.

Materials and methods

Raw materials

The experiments were carried out using apples variety (Golden Delicious) purchased from a local market. Until the processing time, the apples were stored in cold chamber at temperature of 4 °C and relative air humidity of 75%. Before the drying processes the samples were washed, peeled, and sliced from the mesocarp in order to obtain uniform samples with thickness of 3 ± 10^{-1} mm, before being reduced to a cylinder form with diameter of 43 ± 10^{-1} mm. Several measurements were made using a caliper and only samples with a tolerance of ± 5 % were used. The initial moisture content of fresh slices and the final moisture content of dried samples were determined gravimetrically by hot air oven method (oven type LMF-3550/120, Omega, USA) at 105 °C and atmospheric pressure for a period of 24 h.

Drying procedure

The experimental data set of drying kinetics of apple slices were obtained on the experimental set-up, that is designed to imitate an industrial far-infrared vacuum dryer, described by Mitrevski *et al.*, [19]. In drying experiments the required temperature in the vacuum chamber was achieved by regulating of the resistance of the infrared heaters with use on the programmable controller, while the vacuum pressure was achieved by means of single step rotary vane vacuum pump, (type EQ-2XZ, MTI Korea, Korea). The transient temperatures of drying samples were measured with three micro-thermocouples type K, inserted in the mid-plane, ($x = 0$) of each of the three slices of apple. An arithmetical mean of the readings from the three thermocouples was used as a transient temperature reading, $t_x = 0$, for comparison with calculated temperature. The micro-thermocouples were connected to data acquisition system comprised of a computer interface (type IDRN-ST, Omega, USA), and 24-bit A/D converter (type OMB-DAQ-2408, Omega, USA). The measurement of sample's mass changes with time was enabled with load cell (type OMEGA LCL 040, Omega, USA), which was connected to the data acquisition system. The values of the temperature of infrared heaters, t_h and vacuum pressure in vacuum chamber was kept constant during single experiments. In drying experiments the temperature of heaters, were varied between 120-200 °C, while the value of absolute pressure in vacuum chamber were varied between 20-80 kPa. The changes of the temperature of heaters, transient temperatures of drying samples and changes of mass of the drying samples were recorded continuously at one minute time interval on the personal computer. The drying

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was carried out to final moisture content of 0.04 kg/kg d. m. from initial moisture content of 5.69 kg/kg d. m.

A mathematical model of drying

The physical problem involves a single slice of apple of thickness, $2L$, initially at uniform temperature and uniform moisture content, fig. 1.

In the case of an infinite flat plate when the influence of thermodiffusion is small, $\delta = 0$, the unsteady temperature, $t(x, \tau)$, and moisture content, $u(x, \tau)$, fields in the drying body are expressed by the following system of coupled non-linear PDE for energy and moisture transport [10, 20]:
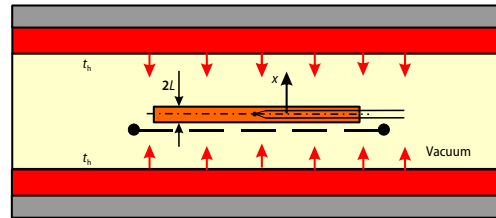

Figure 1. Scheme of drying experiment

$$c\rho_s \frac{\partial t}{\partial \tau} = \lambda \frac{\partial^2 t}{\partial x^2} + \varepsilon r \frac{\partial(\rho_s u)}{\partial \tau} \tag{1}$$

$$\frac{\partial(\rho_s u)}{\partial \tau} = \frac{\partial}{\partial x}\left(a_m \rho_s \frac{\partial u}{\partial x}\right) \tag{2}$$

Here, τ, x, c, λ, ε, r, a_m, and ρ_s are time, normal distance from the mid-plane of the plate, specific heat, thermal conductivity, ratio of water evaporation rate to the reduction rate of the moisture content, latent heat of vaporization, moisture diffusivity, and density of dry solid, respectively.

The shrinkage effect was incorporated through the changes of the specific volume of the drying body. There are several models for describing the changes of the specific volume of the body during drying. In this paper the linear relationship between the specific volume, v_s, and the moisture content, u, was used [10, 20]:

$$v_s = \frac{1}{\rho_s} = \frac{V}{m_s} = \frac{1 + \beta' u}{\rho_{bo}} \tag{3}$$

The problem of the moving boundaries due to the changes of the dimensions of the body during the drying was resolved by introducing the dimensionless co-ordinate:

$$\psi = \frac{x}{L(\tau)} \tag{4}$$

Substituting the expression for ($\rho_s = 1/v_s$) and ψ into eqs. (1) and (2) and rearranging, the resulting system of equations for the temperature and moisture content prediction becomes [20, 21]:

$$\frac{\partial t}{\partial \tau} = \frac{\lambda}{\rho_s c} \frac{1}{L^2} \frac{\partial^2 t}{\partial \psi^2} + \frac{\psi}{L} \frac{\partial L}{\partial \tau} \frac{\partial t}{\partial \psi} + \frac{\varepsilon r}{c} \frac{\rho_s}{\rho_{bo}} \left(\frac{\partial u}{\partial \tau} - \frac{\psi}{L} \frac{\partial L}{\partial \tau} \frac{\partial u}{\partial \psi}\right) \tag{5}$$

$$\frac{\partial u}{\partial \tau} = a_m \frac{\rho_{bo}}{\rho_s} \frac{1}{L^2} \frac{\partial^2 u}{\partial \psi^2} + \left[\frac{\rho_{bo}}{\rho_s^2} \frac{1}{L^2} \frac{\partial(a_m \rho_s)}{\partial \psi} + \frac{\psi}{L} \frac{\partial L}{\partial \tau}\right] \frac{\partial u}{\partial \psi} \tag{6}$$

As initial conditions, uniform temperature and moisture content profiles are assumed:

$$\tau = 0, \quad t(\psi, 0) = t_0, \quad u(\psi, 0) = u_0 \tag{7}$$

The temperature and the moisture content boundary conditions on the surfaces of the drying slice are:

$$-\lambda \frac{1}{L}\left(\frac{\partial t}{\partial \psi}\right)_{\psi=1} + j_q - r(1-\varepsilon)j_{\mathrm{m}} = 0 \tag{8}$$

$$u_{\psi=1} = u_{\mathrm{eq}}(t_{\psi=1}, p) \tag{9}$$

The heat flux, $j_q(\tau)$, and mass flux, $j_{\mathrm{m}}(\tau)$, on the surfaces of drying slice are, [21]:

$$j_q = \varepsilon_r \sigma(T_h^4 - T_{\psi=1}^4) \tag{10}$$

$$j_{\mathrm{m}} = -\rho_s L \frac{d\overline{u}}{d\tau} \tag{11}$$

where ε_r is the emissivity of the material, σ – the Stefan-Boltzman constant, T_h – the temperature of heaters, and $T_{\psi=1}$ – the surface temperature of dried material.

The boundary conditions on the mid-plane of the drying slice are:

$$\left(\frac{\partial t}{\partial \psi}\right)_{\psi=0} = 0, \quad \left(\frac{\partial t}{\partial \psi}\right)_{\psi=0} = 0 \tag{12}$$

The system of non-linear PDE (5) and (6) with initial conditions eq. (7) and boundary conditions, eqs. (8), (9), and (12), and eqs. (10) and (11) is the mathematical model of simultaneous heat and moisture transport within the material and from its surface to the surroundings in case of far-infrared vacuum drying of shrinkage body.

Numerical solution

A complex dependence on thermophysical properties of materials from temperature and moisture content makes the resulting system of PDE non-linear. The same cannot be solved without linearization which leads to unpermitted simplification of the problem. For this reason, the numerical solutions were applied. In order to approximate the solution of eqs. (5) and (6), an explicit numerical procedure has been used. The derivatives with respect to time was represented using forward differencing at the grid point (i, j). All first and second order space derivatives were approximated at time level (j) using central differencing. The values of temperature and moisture content in the first term was assigned its value at the grid point (i, j). Central differencing was also applied to the boundary conditions space derivatives. The number of the space grid points was 101 in all the drying processes calculation scheme. It is defined a network of grid points with grid spacing's:

$$\Delta\psi = \frac{1}{M-1} \Delta\tau = R_{KR}(\Delta\psi)^2 \tag{13}$$

If you insert the replaces:

$$R_{KR} = \frac{R_K}{a_m}, \quad R_{KT} = A_T R_{KR}, \quad \Delta R_{KT} = 0.5 B_T \Delta\psi R_{KR} \tag{14}$$

$$R_{KU} = A_U R_{KR} \quad \Delta R_{KU} = 0.5 B_U \Delta\psi R_{KR} \tag{15}$$

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and if in eqs. (5) and (6) the partial derivatives are replacement with finite differences, will receive the differentiated equations for calculating the value of temperature and moisture content at different points in space networks in the next time level $j + 1$ according to the values of the previous:

$$t_{i,j+1} = (R_{KT} - \Delta_{RKT})t_{i-1,j} + (1 - 2R_{KT})t_{i,j} + (R_{KT} + \Delta_{RKT})t_{i+1,j} \tag{16}$$

$$u_{i,j+1} = (R_{KU} - \Delta_{RKU})u_{i-1,j} + (1 - 2R_{KU})u_{i,j} + (R_{KU} + \Delta_{RKU})u_{i+1,j} \tag{17}$$

A sufficient condition for numerical stability of the explicit method is the values of the ratios, R_{KT} and R_{KU} are less or equal to 0.5:

$$R_{KT} \leq 0.5 \qquad R_{KU} \leq 0.5 \tag{18}$$

For the determination the values of temperature and moisture content on the boundary surface of the dried materials the boundary conditions (8) and (9) transformed in the form of finite differences are used:

$$t_{M+1,j+1} = t_{M-i,j+1} + \frac{2\Delta\psi L}{\lambda}\left[\varepsilon_r\sigma(T_h^4 - T_{M,j+1}^4) + r(1-\varepsilon)\rho_s L\frac{\overline{u}_j - \overline{u}_{j-1}}{\Delta\tau}\right] \tag{19}$$

$$u_{M+1,j+1} = u_{eq}(t_{M+1,j+1};\phi_{M+1,j+1},p) \tag{20}$$

Results and discussion

On the basis of the numerical solutions a computer program TVDShr for calculation of temperature profiles, transient moisture content, mid-plane temperature and the volume averaged moisture content changes was developed.

For the calculation of thermophysical properties the appropriate models with parameters for apple were taken. The moisture diffusivity dependence on moisture and temperature exerts a strong influence on the drying process calculation. Moisture diffusivity of foods is often considered as an Arrhenius-type temperature function

$$a_m(T_k) = a_{m0}\exp\left(-\frac{E_0}{RT_k}\right) \tag{21}$$

In this study our values for the Arrhenius factor, $a_{m0} = 7.913 \cdot 10^{-5}$ m^2/s and activation energy, $E_0 = 36.22$ kJ/molK were utilized [21]. Those values were estimated from experimentally measured temperature response of a drying apple slices by using inverse approach.

The heat capacity apple is equal to the sum of the heat capacity of solid matter and water absorbed by that solid:

$$c = c_s + c_w u \tag{22}$$

Although the heat capacity of solid matter, c_s, and water, c_w, are functions of the temperature, constant values have been most widely used. The following values, proposed in reference [22] for apple, were used: $c_s = 1415$ J/kgK, and $c_w = 4187$ J/kgK.

For practical calculations the system of the two simultaneous PDE could be used by treating the thermal conductivity as constant. A mean value from the results obtained in [23] for the apple, $\lambda = 0.35$ W/mK was utilized in this paper.

The influence of the phase conversion factor ($0 \leq \varepsilon \leq 1$) on the transient moisture content and temperature profiles is very small, and at this reason a mean value, $\varepsilon = 0.5$ was used in the paper.

In numerical calculations the values of density of fully dried body, $\rho_{bo} = 768$ kg/m^3 and the shrinkage coefficient, $\beta' = 0.75$ were used [10, 20].

For calculation of equilibrium moisture content the GAB isotherm equation was used:

$$u_{eq} = \frac{u_m CKa_w}{(1 - Ka_w)(1 - Ka_w + CKa_w)} \tag{23}$$

were the monolayer moisture, u_m, and the adsorption constants, C and, K are related as Arrhenius type equations:

$$u_m = u_x \exp\left(\frac{\Delta H_x}{RT_k}\right), \quad C = C_0 \exp\left(\frac{\Delta H_c}{RT_k}\right), \quad K = K_0 \exp\left(\frac{\Delta H_k}{RT_k}\right) \tag{24}$$

The GAB model parameters, C_0, ΔH_c, K_0, ΔH_k, u_x, and ΔH_c can be estimated by different regression procedures from experimental isotherm data. The experimental results for apple ($u_x = 6.21 \cdot 10^{-2}$ kg/kg; $\Delta H_x = 1.470$ kJ/mol; $C_0 = 0.013$; $\Delta H_c = 12.32$ kJ/mol; $K_0 = 1.380$; $\Delta H_k = -0.920$ kJ/mol) were used in this study, [10].

For verification of a mathematical model a number of numerical calculations were carried out with experimental conditions similar to those in the experiment E11: $t_h = 158.5$--161.5 $^\circ$C, $p = 59.0$-60.5 kPa, $2L_0 = 2.90$-$3.20 \cdot 10^{-3}$ m, $u_0 = 5.60$-5.75 kg/kg, and $t_0 = 24.0$-25.0 $^\circ$C.

In figs. 2 and 3 the temperature profiles and transient moisture content for experimental conditions similar to those in the experiment of far-infrared vacuum dried apple slices (experiment E2.11), [21]: $t_0 = 24.55$ $^\circ$C, $u_0 = 5.69$ kg/kg, $L_0 = 1.48 \cdot 10^{-3}$ m, $t_h = 160$ $^\circ$C, and $p = 60$ kPa are shown.
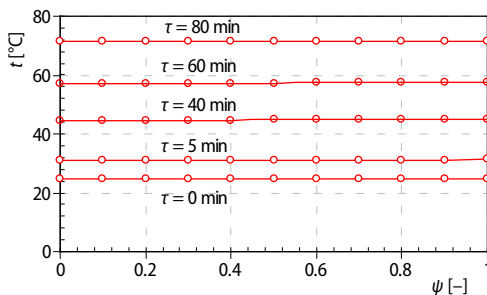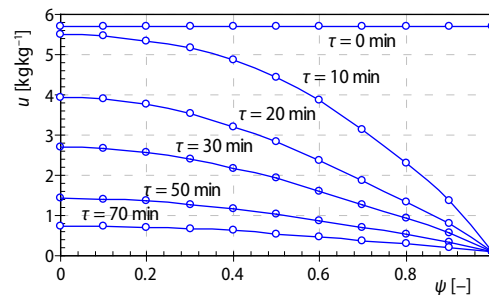

Figure 2. Transient temperature profiles

Figure 3. Transient moisture content profiles

In fig. 4 the experimental transient temperature reading, $t_{x=0}$, and the experimental volume-averaged moisture content, u, change during the far-infrared vacuum drying of a apple slice (experiment E11), are compared with predicted. From fig. 4 is obviously that have very good agreement between the experimental and numerical temperature and moisture content changes, during the drying of the apple slices.

Conclusions

A mathematical model of far-infrared vacuum drying of shrinking bodies was presented. On the basis of the numerical solutions a computer program TVDShr for calculation

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of temperature profiles, transient moisture content, mid-plane temperature, and the volume averaged moisture content changes was developed. From all thermophysical properties the moisture diffusivity has a strong influence on the drying process calculation. In this study our values for the Arrhenius factor and activation energy were utilized. For verification of a mathematical model a number of numerical calculations were carried out with experimental conditions similar to those in the realized real experiment of far-infra red vacuum drying of apple slices. The very good agreement between

Figure 4. Mid-plane temperature, *t*, temperature of the heaters, *t_h*, and volume-averaged moisture content, *u*, changes during the drying of a apple slice

the experimental and numerical temperature and volume-averaged moisture content changes during the far-infra red vacuum drying of apple slices showed that the developed mathematical model it would be a useful tool for engineering and scientific calculations, and design and optimization of far-infra red vacuum drying processes.
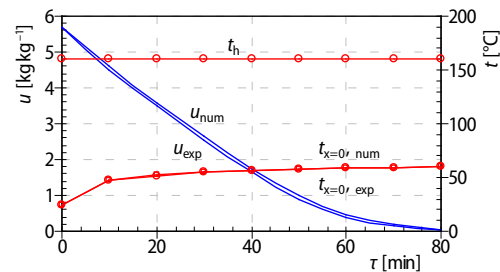
Nomenclature

a_m – moisture diffusivity, [m^{2}s^{-1}]
a_{m0} – Arrhenius factor, [m^{2}s^{-1}]
a_w – water activity, [–]
c – heat capacity (dry basis), [JK^{-1}kg^{-1} d. b.]
C_0, K_0 – GAB model parameters, [–]
E_0 – activation energy, [Jkg^{-1}]
$\Delta H_c, \Delta H_k, \Delta H_x$ – GAB model parameter, [Jmol^{-1}]
j_m – mass flux, [kgm^{-2}s^{-1}]
j_q – heat flux, [Wm^{-2}]
L – flat plate thickness, [m]
M – space of grid point, [–]
m_s – mass of dry solid, [kg]
R – absolute gas constant, [JK^{-1}mol^{-1}]
p – pressure, [Pa]
r – latent heat of vaporization, [Jkg^{-1}]
t – temperature, [°C]
T_k – temperature, [K]
v – specific volume, [m^{3}kg^{-1}]
V – volume, [m^3]
x – distance from the mid-plane, [m]
u – moisture content, [kg^{-1}kg^{-1} d. b.]

Greek symbols

β' – shrinkage coefficient, [–]
δ – thermo-gradient coefficient, [K^{-1}]
ε – phase conversion factor, [–]
ε_r – emissivity of the material, [–]
λ – thermal conductivity, [Wm^{-1}K^{-1}]
ρ – density, [kgm^{-3}]
σ – Stefan-Boltzman constant, [Wm^{-2}K^{-4}]
τ – time, [s]
ψ – dimensionless co-ordinate, [–]

Subscripts

bo – fully dried body
eq – equilibrium
h – heater
m – monolayer
0 – initial
w – water
s – dry solid

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